Mathematical
2025 – 2024
prep 1
Second term
Student name

Lesson 1 Powers and Exponents

Repeated multiplication and exponential form

⊙ * The product of repeated factors can be expressed in powers or base form, i.e. using a base and a base.

(Exponent) shows the number of times the basis is repeated

(The base) is the recurring factor.

Notice that: $3^4 = 3\times3\times3\times3$

⊙ 3^4 It is read as "3 to the power of 4", which means "3 multiplied by itself 4 times".

For example

⊙ $625 = (-5)(-5)(-5)(-5) = (-5)^4$

⊙ $\frac{16}{81} = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \left(\frac{2}{3}\right)^4$

⊙ $32 = 2\times2\times2\times2\times2 = (2)^5$

⊙ $-x^3 = (-x)\times(-x)\times(-x) = (-x)^3$

⊙ $\frac{1}{4} = \frac{-1}{2}\times\frac{-1}{2} = \left(\frac{-1}{2}\right)^2$

⊙ $25 = 5\times5 = (5)^2$

Notice that

⊙ $(5)^2$ It is the exponential form of the number 25

⊙ $\left(\frac{1}{2}\right)^3$ It is the exponential form of the number $\frac{1}{8}$

⊙ $(-3)^3$ It is the exponential form of the number -27

.Write each of the following using exponents

Example 1

(1) $3\times3\times3\times3\times3\times3\times3$	(2) $(-5)\times(-5)\times(-5)$	(3) $2\times2\times2\times3\times3$
3^7	$(-5)^3$	$(2)^3\times(3)^2$
$a\times a\times5\times5\times5$ (4)	(5) $7\times7\times x\times x\times7$	(6) $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$
$(a)^2\times(5)^3$	$7^3\times x^2$	$\left(\frac{1}{2}\right)^5$

exercise

.Write each of the following using exponents

(1) $8\times8\times8\times8\times8\times8$

(2) $(-4)\times(-4)\times(-4)\times(-4)$

(3) $7\times7\times5\times5\times5$

(4) $x\times x\times x\times x\times5\times5$

(5) $a\times a\times a\times5\times5\times5\times a\times a$

(6) $\left(\frac{-1}{2}\right)\times\left(\frac{-1}{2}\right)\times\left(\frac{-1}{2}\right)$

Solution

(1) .... (2) .... (3) ....

(4) .... (5) .... (6) ....

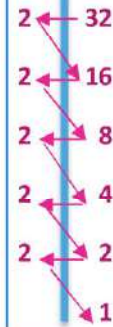
Example 2

Write each of the following in exponential form so that the base is a prime number

(1) 108

(2) 32

$108 = 3\times3\times3\times2\times2 = 3^3\times2^2$

Solution

$32 = 2\times2\times2\times2\times2 = 2^5$

Solution

Exercise

Write each of the following in exponential form so that the base is a prime number

(1) 216

(2) 100

(3) 81

(1)..

(2)..

(3)..

example 3

If: a=3 , b=-2, find the numerical value of each of the following

(1) $5a^2$ (2) $2b^3$ (3) $(3b)^2$ (4) b^a

(5) $(a+b)^5$ (6) a^2+b^2

solution

(1) $5a^2=5\times(3)^2=5\times9=45$

(2) $2b^3=2\times(-2)^3=2\times-8=-16$

(3) $(3b)^2=(3\times(-2))^2=(-6)^2=36$

(4) $b^a=(-2)^3=-8$

(5) $(a+b)^5=(3+(-2))^5=(1)^5=1$

(6) $a^2+b^2=(3)^2+(-2)^2=9+4=13$

Exercise

If : x=4 , y=-5, find the numerical value of each of the following

(1) $(2y)^2$	(2) $3x^2$	(3) $2y^3$
(4) y^x	(5) $(y-x)^2$	(6) $(x+y)^3$

solution

(1)	(2)	(3)
(4)	(5)	(6)

Even and odd exponents of negative base

⊙ If the base is a negative number and the exponent is an even number, the result is a positive number .

⊙ If the base is a negative number and the exponent is an odd number, the result is a negative number.

$\odot\ (-3)^4=-3\times-3\times-3\times-3=81$

$\odot\ (-3)^3=-3\times-3\times-3=-27$

Multiply and divide powers with the same base

Multiplication law

When multiplying powers that have the same base, we keep the base and add the exponents.

$3^2 \times 3^4 = 3^{2+4} = 3^6$

For any rational number a, and two integers n and m

$a^m \times a^n = a^{m+n}$

Division law

When dividing powers that have the same base, we keep the base and subtract the exponents.

$3^7 \div 3^5 = 3^{7-5} = 3^2$

For any rational number a, and two integers n and m

$a^m \div a^n = a^{m-n}$

Note

The multiplication law can be generalized to more than two powers with the same base:

For example : $2^3 \times 2^2 \times 2 = 2^{3+2+1} = 2^6 = 64$

Example 4

Simplify each of the following

$(1)\ 3^4 \times 3$	$(2)\ (-2)^2 \times (-2) \times (-2)^3$	$(3)\ (-3)^4 \times (3)$	$(4)\ \left(\frac{1}{5}\right) \times \left(\frac{1}{5}\right)^3$
$(5)\ (-2)^5 \times 2^3 \times 2$	$(6)\ \frac{7^5}{7^2}$	$(7)\ (-6)^3 \div (-6)^2$	$(8)\ \left(\frac{2}{3}\right)^5 \div \left(\frac{2}{3}\right)^2$

Solution

$(1)\ 3^{4+1} = 3^5$	$(2)\ (-2)^{2+1+3} = (-2)^6 = 2^6$	$(3)\ (3)^4 \times 3 = 3^5$	$(4)\ \left(\frac{1}{5}\right)^{1+3} = \left(\frac{1}{5}\right)^4$
$(5)\ -(2)^5 \times 2^3 \times 2 = = -(2)^{5+3+1} = -5^9$	$(6)\ 7^{5-2} = 7^3$	$(7)\ (-6)^{3-2} = (-6)^1 = -6$	$(8)\ \left(\frac{2}{3}\right)^{5-2} = \left(\frac{2}{3}\right)^3$

Exercise

Simplify each of the following

$(1)\ 2^3 \times 2^2 \times 2$	$(2)\ \left(\frac{3}{5}\right)^4 \div \left(\frac{3}{5}\right)^3$	$(3)\ (-2)^2 \times (-2)^5$	$(4)\ 5^3 \times 5^2$
$(5)\ \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^2 \times \left(\frac{-1}{2}\right)^4$	$(6)\ (-3)^3 \times (-3)^5 \times (3)^2$	$(7)\ (-3)^5 \div (3)^2$	$(8)\ (-2)^4 \times 2^3$

Solution

(1)	(2)	(3)	(4)
(5)	(6)	(7)	(8)

Example 4

Simplify each of the following

(1) $\frac{2^3 \times 2^5}{2^7}$	(2) $\frac{5^5 \times 5^6}{5^8 \times 5}$	(3) $\frac{3^4 \times 3^3}{3 \times 3^5}$	(4) $\frac{(-2)^7 \times 3^6}{(-2)^5 \times 3^4}$
(5) $\frac{6^5 \times 6^4 \times 6^3}{6^{12}}$	(6) $\frac{(-7)^2 \times 7^8}{(-7) \times (-7)^7}$	(7) $\frac{a^4 \times a^5}{a^6 \times a^2}$	(8) $\frac{(-x)^4 \times x^6}{(-x)^5 \times (-x)^3}$

Solution

(1) $\frac{2^{3+5}}{2^7} = \frac{2^8}{2^7} = 2^1 = 2$

(2) $\frac{5^{5+6}}{5^{8+1}} = \frac{5^{11}}{5^9} = 5^2 = 25$

(3) $\frac{3^{4+3}}{3^{1+5}} = \frac{3^7}{3^6} = 3^1 = 3$

(4) $= (-2)^{7-5} \times (3)^{6-4}$
$= (-2)^2 \times (3)^2 = 4 \times 9 = 36$

(5) $\frac{6^{5+4+3}}{6^{11}} = \frac{6^{12}}{6^{11}} = 6^1 = 6$

(6) $\frac{(7)^2 \times 7^8}{-(7) \times -(7)^7}$
$= \frac{(7)^2 \times 7^8}{(7) \times (7)^7} = \frac{7^{2+8}}{7^{1+7}}$

(7) $\frac{a^{4+5}}{a^{6+2}} = \frac{a^9}{a^8} = a^1 = a$

(8) $= \frac{=(x)^4 \times x^6}{-(x)^5 \times -(x)^3}$
$= \frac{x^{4+6}}{x^{5+3}} = \frac{x^{10}}{x^8} = x^2$

Exercise

Simplify each of the following

(1) $\frac{7^5 \times 7^7}{7^{11}}$	(2) $\frac{6^6 \times 6^5}{6^3 \times 6^7}$	(3) $\frac{(-2) \times (-2)^9}{(-2)^3 \times (-2)^5}$	(4) $\frac{x^3 \times x^4 \times x^5}{x^{10}}$
(5) $\frac{(-a)^6 \times a^7}{(-a)^4 \times (-a)^7}$	(6) $\frac{b^3 \times b \times b^7}{b^2 \times b^6}$	(7) $\frac{3^6 \times (-5)^5}{3^5 \times (-5)^3}$	(8) $\frac{(-2)^4 \times (-2) \times 2^6}{2^7}$

Home work 1

Q1 .Choose the correct answer from the given answers

1. Which of the following is equal to $(-3)^3$
   (a) 9 (b) -9 (c) 27 (d) -27

2. Which of the following is equal to $5\times5\times5\times5\times5\times5$
   (a) 5×6 (b) 6^5 (c) 5^6 (d) $5+6$

3. Which of the following is equal to -5^4
   (a) 20 (b) -20 (c) -625 (d) 625

4. Which of the following is equal to $(-5)^4$
   (a) 20 (b) -20 (c) 625 (d) -625

5. Which of the following is equal to -5^4
   (a) 20 (b) -20 (c) -625 (d) 625

6. If $3^2\times a=3^8$,then a equal ....
   (a) 2^{16} (b) 2^4 (c) 2^6 (d) a^4

7. The multiplicative inverse of a number $(-1)^3$ is ?
   (a) $(-1)^3$ (b) $(-1)^2$ (c) 1^2 (d) 1^3

8. Additive inverse of a number $(-2)^2$ is ?
   (a) 2^2 (b) 4 (c) -4 (d) 0

Q2 Simplify each of the following

1. $\frac{(-3)^4\times(-3)^7}{(-3)^3\times(-3)^6}$
2. $\frac{5^3\times5^4\times5^5}{5^{10}}$
3. $\frac{2^5\times2^6}{2^{10}}$
4. $\frac{a^6\times a^7\times a}{a^{10}}$
5. $\frac{(-5)^3\times(5)^2}{(-5)^4}$
6. $\frac{(-2)^5\times(4)^7}{(-2)^4\times(4)^5}$

Q3 Using prime factors and exponents, write each of the following numbers

1. 324
2. 400
3. 225
4. 144
5. 125
6. 64

Q3 :Find the numerical values of each of the following quantities at the given values

1. $(a-b)^d$ when $a=2$, $b=3$, $d=4$
2. $(-a)^b$ when $a=-5$, $b=3$
3. $a(b-c^d)$ when $a=2$, $b=3$, $d=3$
4. b^c+c^d when $b=3$, $c=2$, $d=4$

Minor powers and negative integer powers

⊙ By observing the pattern in the table, we find that:

$3^0 = 1$ ، $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$ ، $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ ، $3^{-1} = \frac{1}{3}$

3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$

÷3 ÷3 ÷3 ÷3 ÷3 ÷3

Notice that

(1) Any number not equal to zero raised to the power of "zero" equals "1."

If $a \neq 0$ then $(a^0 = 1)$, for example : $(-5)^0 = 1$ ، $\left(\frac{1}{3}\right)^0 = 1$ ، $2^0 = 1$

(2) Any number not equal to zero raised to the power $(-n)$ equal The multiplicative inverse of a number raised to the power is equal to (n)

If $a \neq 0$ then $a^{-n} = \frac{1}{a^n}$, for example $\left(\frac{2}{7}\right)^{-1} = \frac{7}{2}$ ، $\left(\frac{1}{5}\right)^{-3} = 5^3$ ، $3^{-2} = \frac{1}{3^2}$

Notice

(3) a^0 It is the multiplicative neutral number because $(a^0 = 1)$

(4) Each of a^n , a^{-n} is a multiplicative inverse for the other in because $(a^n \times a^{-n} = a^0 = 1)$

Example 1

Simplify each of the following

(1) $\left(1\frac{2}{3}\right)^{-2}$

$\left(\frac{5}{3}\right)^{-2} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$

(2) $\left(\frac{3}{5}\right)^{-1}$

$\left(\frac{5}{3}\right)^1 = \frac{5}{3}$

(3) 3^{-2}

$\frac{1}{3^2} = \frac{1}{9}$

(4) 5^{-1}

$\frac{1}{5}$

(5) $2^{-5} \times 2^3$

$2^{-5+3} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

(6) $7^3 \times 7^{-3}$

$7^{3+(-3)} = 7^0 = 1$

(7) $4^{-2} \times 4^5$

$4^{-2+5} = 4^3 = 64$

(8) $(0.5)^{-2}$

$\left(\frac{5}{10}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

(9) $\left(\frac{1}{2}\right)^{-1} \div \frac{1}{2}$

(10) $(-2)^{-2} \div (2)^{-4}$

(11) $\frac{5^{-2}}{5^2}$

(12) $5^3 \div 5^5$

$\left(\frac{1}{2}\right)^{-1-1}=\left(\frac{1}{2}\right)^{-2}=2^2=4$	$2^{-2-(-4)}=2^2=4$	$5^{-2-2}=5^{-4}=\frac{1}{5^4}=\frac{1}{625}$	$3^{3-5}=3^{-2}=\frac{1}{3^2}=\frac{1}{9}$

Exercise

Simplify each of the following

$(1)\left(2\frac{1}{3}\right)^{-2}$	$(2)\left(\frac{1}{2}\right)^{-3}$	$(3)7^{-1}$	$(4)3^{-3}$
$(5)\frac{5^{-3}}{5}$	$(6)\left(\frac{3}{5}\right)^{-3}\div\left(\frac{3}{5}\right)^{-2}$	$(7)\left(\frac{3}{2}\right)^{3}\times\left(\frac{2}{5}\right)^{0}$	$(8)7^{-5}\times7^{4}$

Example

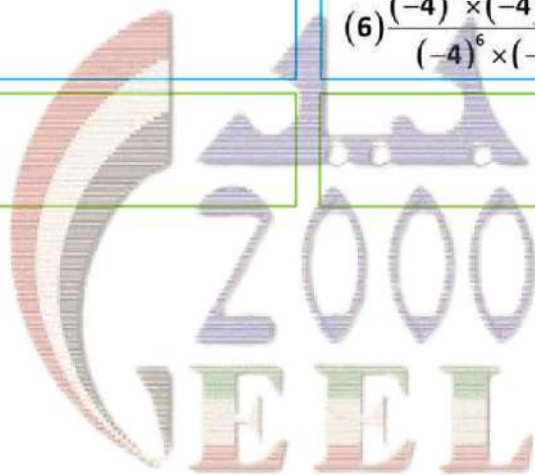
Simplify each of the following

$(1)\frac{3^5\times3^{-3}}{3^4}$	$(2)\frac{2^5\times2^{-2}}{2^3\times2^{-4}}$
$\frac{3^{5-3}}{3^4}=\frac{3^2}{3^4}=3^{2-4}=3^{-2}=\frac{1}{3^2}=\frac{1}{9}$	$\frac{2^{5+(-2)}}{2^{3+(-4)}}=\frac{2^3}{2^{-1}}=2^{3-(-1)}=3^{3+1}=3^4=81$
$(3)\frac{a^{-6}\times a^{-2}}{a^{-3}\times a^{-4}}$	$(4)\frac{5^2\times5^{-3}\times5^4}{5^3}$
$\frac{a^{-6+(-2)}}{a^{-3+(-4)}}=\frac{a^{-8}}{a^{-7}}=a^{-8-(-7)}=a^{-1}=\frac{1}{a}$	$\frac{5^{2+(-3)+4}}{5^3}=\frac{5^3}{5^3}=5^{3-3}=5^0=1$
$(5)\frac{7^2\times7^{-4}}{7^{-2}}$	$(6)\frac{x^{-1}\times x^{-3}\times x^2}{x^{-7}\times x^4}$
$\frac{7^{2+(-4)}}{7^{-2}}=\frac{7^{-2}}{7^{-2}}=7^{-2-(-2)}=7^0=1$	$\frac{x^{-1+(-3)+2}}{x^{-7+4}}=\frac{x^{-2}}{x^{-3}}=x^{-2+3}=x^1=x$

2

Exercise

Simplify each of the following

(1) $\frac{3^2 \times 3^{-1} \times 3^5}{3^5}$	(2) $\frac{2^{-3} \times 2^{-5}}{2^{-6}}$
(3) $\frac{3^4 \times 3^2 \times 3}{3^8}$	(4) $\frac{5^{-2} \times 5^3}{5 \times 5^4}$
(5) $\frac{a^{-2} \times a^3 \times a^{-4}}{a^2 \times a}$	(6) $\frac{(-4)^4 \times (-4)^3 \times 4^2}{(-4)^6 \times (-4)^5}$

Home work — **Minor powers and negative integer powers**

Q1 .Choose the correct answer from the given answers

1 Which of the following is equal to $a^{-3} \times a$?

(a) $\frac{1}{a^2}$ (b) $\frac{1}{a^3}$ (c) a^2 (d) a^4

2 Which of the following is equal to $x^{-1} \times x^3$?

(a) x^4 (b) x^2 (c) $\frac{1}{x^3}$ (d) $\frac{1}{x^2}$

3 Which of the following is equal to -2^4 ?

(a) -16 (b) -8 (c) $\frac{1}{16}$ (d) $\frac{-1}{16}$

4 Which of the following is equal to 3^{-4} ?

(a) -81 (b) 81 (c) $\frac{1}{12}$ (d) $\frac{1}{81}$

5 Which of the following is the additive inverse of the number? 4^{-3} ?

(a) $(-4)^3$ (b) $(-4)^{-3}$ (c) 4^3 (d) 4^{-3}

6 If $2^4 \times a = 2^2$, what is the value of a ?

(a) 2^8 (b) 2^2 (c) 2^{-2} (d) 2

7 $5a^0 - (5a)^0 = ...$

(a) 0 (b) 4 (c) 5 (d) 10

8 Which of the following is the multiplicative inverse of the number 4^{-3}

(a) $(-4)^3$ (b) $(-4)^{-3}$ (c) 4^3 (d) 4^{-3}

9 If $a^x \times a^7 = a^9$, then x can be equal to .....

(a) 16 (b) -2 (c) 2 (d) -16

10 Which of the following expresses the quantity $\frac{a^{-3}}{a^{-7}}$ in simplest form?

(a) $\frac{1}{a^4}$ (b) a^4 (c) a^8 (d) $\frac{1}{a^8}$

11 If $c^{-3} \times c^x = 1$, then x can be equal to ....

1 (d) -3 (c) 3 (b) 0 (a)

12 If $\frac{b^x}{b^4} = b^3$, then x can be equal to ....

(a) 12 (b) 1 (c) -1 (d) 7

13 Which of the following equal to $2^a \times 2^a$?

(a) 4^{2a} (b) 2^a (c) 2^{a+1} (d) 2^{2a}

14 Which of the following equal to $2^a + 2^a$?

(a) 4^{2a} (b) 2^a (c) 2^{a+1} (d) 2^{2a}

15 Which of the following is equal to a quarter of the number 2^x ?

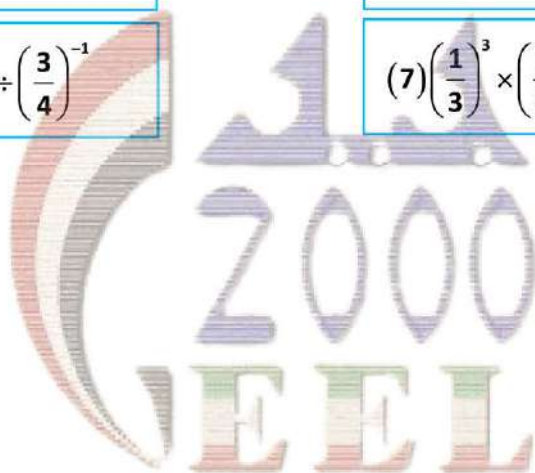
2^{x-1} (c) 2^{x-2} (d) $\left(\frac{1}{2}\right)^x$ (b) 1^x (a)

16 Which of the following is equal to half the number 2^x ?

2^{x-1} (c) 2^{2x} (d) $\left(\frac{1}{2}\right)^x$ (b) 1^x (a)

Q2	Simplify each of the following
(1) $\frac{5^2 \times 5^{-1} \times 5^5}{5^5}$	(2) $\frac{3^{-4} \times 3^{-5}}{3^{-6}}$
(3) $\frac{2^5 \times 2^3 \times 2}{2^{10}}$	(4) $\frac{6^{-3} \times 6^8}{6^2 \times 6^5}$
(5) $\frac{x^{-2} \times x^3 \times x^{-4}}{x \times x^2}$	(6) $\frac{(-3)^4 \times (3)^3 \times 3^2}{(-3)^6 \times (-3)^5}$

Q3	Simplify each of the following		
(1) $\left(1\frac{1}{2}\right)^{-3}$	(2) $\left(\frac{2}{3}\right)^{-3}$	(3) 5^{-1}	(4) 4^{-2}
(5) $\frac{3^{-3}}{3^{-1}}$	(6) $\left(\frac{3}{4}\right)^{-3} \div \left(\frac{3}{4}\right)^{-1}$	(7) $\left(\frac{1}{3}\right)^3 \times \left(\frac{3}{2}\right)^0$	(8) $4^{-5} \times 5^3$

Lesson 2 Scientific formula

Scientific formula

- It is a way of writing very large or very small numbers, in which the number is written as the product of two factors, one of which has an absolute value greater than or equal to "1" and less than "10" and the other one of the powers of the number 10
- That is: the number in its scientific form is written in the form: $a\times10^n$: $1\le|a|<10$ ، $n\in\mathbb{Z}$

For example Each of the following numbers is written in scientific form

- 1×10^{22}
- -3.15×10^{-3}
- 2.6×10^{5}

write numbers in scientific form

(1) To write the number 0. 000 79 in scientific form

- 0.00079

We move the decimal point 4 digits to the right where $(1\le7.9<10)$ Then we divide the result by 10^4 That is, we multiply by 10^{-4}

Then : $4\ 650\ 000=7.9\times10^{-4}$

(2) To write the number 4 650 000 in scientific form

- 4650000.0

We move the decimal point 6 digits to the left where $(1\le4.65<10)$ Then we multiply it by 10^6

Then : $4\ 650\ 000=4.65\times10^6$

(3) To write the number 0.032×10^5 in scientific form

- We move the decimal point 2 places to the right and then multiply the result by 10^{-2}

$0.032\times10^5=3.2\times10^{-2}\times10^5=3.2\times10^3$

(4) To write the number 571×10^9 in scientific form

- We move the decimal point 2 places to the left and then multiply the result by 10^2

$571\times10^9=5.71\times10^2\times10^9=5.71\times10^{11}$

Example 1 Write down each of the following using the scientific formula

(1) 0.000 000 245

We move the decimal point 7-digit to the right and multiply the result by 10^{-7}

$0.000\ 000\ 245=2.45\times10^{-7}$

(2) 7 100 000 000

We move the decimal point 9-digit to the left and multiply the result by 10^9

$7\ 100\ 000\ 000=7.1\times10^9$

(3) 706.4×10^{5}

We move the decimal point 2-digit to the left and multiply the result by 10^{2}

$706.4\times10^{5}=7.064\times10^{2}\times10^{5}=7.064\times10^{7}$

(4) 45×10^{8}

We move the decimal point 1 place to the left and then multiply the result by 10^{1}

$45\times10^{8}=7.1\times10^{9}$

(5) 0.048×10^{7}

We move the decimal point 2-digit to the right and multiply the result by 10^{-2}

$0.048\times10^{7}=4.8\times10^{7}\times10^{-2}=4.8\times10^{5}$

(6) 0.15×10^{-9}

We move the decimal point 1 place to the right and then multiply the result by 10^{-1}

$0.15\times10^{-9}=1.5\times10^{-9}\times10^{-1}=1.5\times10^{-10}$

Exercise

Write down each of the following using the scientific formula

(1) 0.524

(2) 1320000000

(3) 500000000

(4) -0.000128

(5) 0.00564

(6) -70000000000

(7) 0.054×10^{6}

(8) 132×10^{-4}

(9) 13×10^{8}

Writting numbers in standard form

To convert the number $a\times10^{n}$ from scientific form to standard form

- When n is positive, move the decimal point n of the digits to the right
- When n is negative, move the decimal point $|n|$ of the digits to the left

Example 2

Write each of the following in standard form

(1) 2.56×10^{-2}

Move the decimal point to the left 2 digits after putting zeros on the left 0.0256

(2) 3.5×10^{3}

Move the decimal point to the right 3 digits after placing zeros on the right 3500

Exercise

Write each of the following in standard form

(1) 5.67×10^{-1}	(2) 3.2×10^{-3}	(3) -1.256×10^{5}	(4) 2.53×10^{3}

Example 3

Arrange each of the following in ascending order

4.87×10^{24} ، 3.3×10^{23} ، 5.97×10^{24} ، 6.4×10^{23}

- **First Step:** We arrange the numbers in ascending order according to the exponents
- **Second step :**

4.87×10^{24}, 5.97×10^{24} > 3.3×10^{23}, 6.4×10^{23}

When exponents are equal
$5.97 > 4.87$
$5.97\times10^{24} > 4.87\times10^{24}$

When exponents are equal
$6.4 > 3.3$
$6.4\times10^{23} > 3.3\times10^{23}$

$5.97\times10^{24} > 4.87\times10^{24} > 6.4\times10^{23} > 3.3\times10^{23}$

Exercise

Arrange each of the following in descending order

2.36×10^{12} ، 5.4×10^{14} ، 6.23×10^{12} ، 4.5×10^{14}

Operations on numbers in the scientific formula

Example 3

Find the result of each of the following scientific formula

(1) $(4.5\times10^{-5})\div(1.5\times10^{-3})$

$(4.5\times10^{-5})\div(1.5\times10^{-3})=(4.5\div1.5)\times(10^{-5}\div10^{-3})$
$=3\times10^{-2}$

(2) $(2.5\times10^{5})\times(6\times10^{8})$

$(2.5\times10^{5})\times(6\times10^{8})=(2.5\times6)\times(10^{5}\times10^{8})$
$=15\times10^{13}=1.5\times10^{14}$

(3) $(3.21\times10^{13})-(8.1\times10^{12})$

$(3.21\times10^{13})-(8.1\times10^{12})=(32.1\times10^{12})-(8.1\times10^{12})$
$=(32.1-8.1)\times10^{12}$
$=24\times10^{12}=2.4\times10^{13}$

(4) $(3.1\times10^{2})+(2.5\times10^{3})$

$(3.1\times10^{2})+(2.5\times10^{3})=(3.1\times10^{2})+(25\times10^{2})$
$=(3.1+25)\times10^{2}$
$=28.1\times10^{2}=2.81\times10^{3}$

Exercise

Find the result of each of the following scientific formula

(1) $(2.4\times10^{11})\div(1.2\times10^{-4})$

(2) $(1.2\times10^{5})\times(4\times10^{3})$

(3) $(7.8\times10^{10})-(2.5\times10^{9})$

(4) $(3.6\times10^{3})+(7.4\times10^{4})$

Home work 3

Q1 Choose the correct answer from the given answers

1 Which of the following numbers is not in scientific form ?

(a) 2.35×10^{7} (b) 3.5×10^{-3}
(c) 2.35×10^{-7} (d) 23.5×10^{6}

2 Which of the following numbers is written in scientific form ?

(a) 31.5×10^{6} (b) 15×10^{5}
(c) $1.5\times10^{4.5}$ (d) 3.15×10^{5}

3 Which of the following numbers is not in scientific form ?

(a) 2.4×10^{8} (b) 2.4×10^{-9}
(c) 0.24×10^{-5} (d) 9.7×10^{4}

4 Which of the following numbers is written in scientific form ?

(a) 0.67×10^{5} (b) 0.00053
(c) 60.7×10^{5} (d) 6.7×10^{5}

5 Which of the following is equal to 0.000073?

(a) 7.3×10^{6} (b) 7.3×10^{5}
(c) 7.3×10^{-5} (d) 7.3×10^{-6}

6 Which of the following expresses the number 8 million in scientific form?

(a) 8×10^{7} (b) 8×10^{6}
(c) 8×10^{-6} (d) 8×10^{8}

7 If the number $y\times10^{-9}$ is written in scientific form, which of the following could be the value of y?

(a) 600 (b) 60 (c) 6 (d) 0.6

8 If $6.3\times10^{n}=0.00063$ what is the value of n?

(a) 4 (b) 3 (c) -4 (d) -3

9 The scientific formula of the number 8000×500 is equal to

(a) 0.4×10^{3} (b) 4×10^{5} (c) 400×10^{2} (d) 4×10^{6}

10 What is the value of 2.37×10^{-4}

(a) 0.000237 (b) 0.00237 (c) 23700 (d) 0.0000237

11 If $0.000035=3.5\times10^{n}$, what is the value of n ?

(a) -6 (b) -4 (c) 5 (d) -5

12 Which of the following is equal to a quarter of a million?

(a) 0.25×10^{6} (b) 2.5×10^{9} (c) 2.5×10^{5} (d) 0.25×10^{7}

Q2 Write each of the following numbers in scientific form

1	0.25B	2	5M	3	0.000027	4	36 200 000
5	0.056×10^{-3}	6	12.3×10^{-5}	7	0.35×10^{6}	8	360×10^{11}

Q3 Write each of the following numbers in standard form

1	1.24×10^{-2}	2	2.37×10^{5}	3	5.67×10^{-3}	4	2.3×10^{4}

Q4 Write each of the following numbers in standard form

1. $(9.8\times10^{-5})+(4.9\times10^{-6})$
2. $(4.8\times10^{-7})\div(0.8\times10^{5})$
3. $(4.5\times10^{7})\times(4\times10^{8})$
4. $(9.7\times10^{-5})+(1.27\times10^{-4})$
5. $(2.4\times10^{5})-(4.2\times10^{4})$

Q5 Arrange each of the following in ascending order

1. 0.37×10^{7} ، 3.4×10^{6} ، 4 300 000
2. 0.537×10^{13} ، 6.9×10^{12} ، 73×10^{11}

Q6 Arrange each of the following in descending order

1. 2.5×10^{13} ، 3.6×10^{12} ، 7.8×10^{13} ، 9.6×10^{12}
2. 8.6×10^{7} ، 1.4×10^{7} ، 2.1×10^{7} ، 1.69×10^{8}

Lesson 3 **Square roots and cube roots**

Notice that

⊙ The squares of integers $(\pm1)^2$ ، $(\pm2)^2$ ، $(\pm3)^2$ ، $(\pm4)^2$ ، $(\pm5)^2$ ، .... are equal to 1 ، 4 ، 9 ، 16 ، 25 ، ...... called perfect squares

First Square Root of a whole square

⊙ The square root of a perfect square number (a) is the number whose square is equal to (a)

⊙ A perfect square number has two square roots, one positive and the other negative "each is the addition of the other"

⊙ For example, the number 9 has two square roots, which are:-3 , 3 because $(-3)^2 = 9$ ، $(3)^2 = 9$

Notice that

⊙ The symbol $\sqrt{\ }$ expresses the positive square root of a number, for example: The positive square root of the number 25 is 5 and it is written: $\sqrt{25} = 5$

⊙ The symbol $-\sqrt{\ }$ expresses the negative square root of a number, for example: The positive square root of the number 36 is -6 and it is written: $-\sqrt{36} = -6$

⊙ The symbol $\pm\sqrt{\ }$ expresses the two square roots of a number, for example: two square roots of the number 49 are ±7 and it is written: $\pm\sqrt{49} = \pm7$

Important Notes

(1) $\sqrt{0} = 0$

(2) It makes no sense to find the square root of a negative number

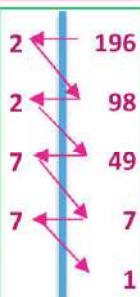
(3) $\sqrt{a^2} = |a|$ for example $\sqrt{(-5)^2} = |-5| = 5$

(4) $\sqrt{a^{2n}} = |a^n|$ for example $\sqrt{x^8} = |x^4|$

Example 1

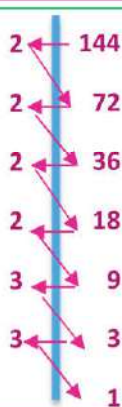
Find each of the following in its simplest form

(1) $\sqrt{196}$

2	196
2	98
7	49
7	7
	1

$196 = 2\times2\times7\times7$

$\sqrt{196} = 2\times7 = 14$

(2) $\sqrt{144}$

2	144
2	72
2	36
2	18
3	9
3	3
	1

$144 = 2\times2\times2\times2\times3\times3$

$\sqrt{144} = 2\times2\times3 = 12$

(3) $\sqrt{(-9)^2}$

$|-9| = 9$

(4) $\sqrt{2.25}$

$\sqrt{\frac{225}{100}} = \frac{15}{10} = \frac{3}{2} = 1.5$

(5) $\pm\sqrt{0.49}$

$\pm\sqrt{\frac{49}{100}} = \pm\frac{7}{10} = \pm0.7$

(6) $-\sqrt{\frac{16}{25}}$

$-\frac{4}{5}$

(7) $\sqrt{(10)^2-(6)^2}$

$\sqrt{100-36} = \sqrt{64} = 8$

(8) $\sqrt{(3)^2+(4)^2}$

$\sqrt{9+16} = \sqrt{25} = 5$

(9) $\sqrt{36+64}$

$\sqrt{100} = 10$

(10) $\sqrt{2\frac{1}{4}}$

$\sqrt{\frac{9}{4}} = \frac{3}{2}$

Exercise

Find each of the following in its simplest form

(1) $\sqrt{225}$	(2) $-\sqrt{81}$	(3) $\pm\sqrt{0.25}$
(4) $-\sqrt{\frac{25}{81}}$	(5) $\pm\sqrt{1\frac{9}{16}}$	(6) $\pm\sqrt{\left(-\frac{2}{5}\right)^2}$
(7) $\sqrt{(12)^2+(5)^2}$	(8) $\sqrt{225-81}$	(9) $\pm\sqrt{9a^4}$

solving equations using square root

If $x^2 = a$ where $a \geq 0$ then $x = \pm\sqrt{a}$:

for example : If $x^2 = 36$ then $x = \pm\sqrt{36} = \pm 6$:

Example

Find the solution set for each of the following equations in Z

(1) $X^2 = 49$

$X = \pm\sqrt{49} = \pm 7$

$S.S. = \{7 ، -7\}$

(2) $X^2 + 1 = 26$

$\because X^2 = 26 - 1$ $\therefore X^2 = 25$

$\therefore X = \pm\sqrt{25} = \pm 5$

$S.S. = \{5 ، -5\}$

(3) $X^2 - 5 = 31$

$\because X^2 = 36 + 5$ $\therefore X^2 = 36$

$\therefore X = \pm\sqrt{36} = \pm 6$

$S.S. = \{6 ، -6\}$

2

(4) $3X^2 - 1 = 74$

$\because 3X^2 = 74 + 1$ $\therefore 3X^2 = 75$

$\therefore X^2 = \frac{75}{3}$ $\therefore X^2 = 25$

$\therefore X = \pm\sqrt{25} = \pm 5$

$S.S. = \{5 ، -5\}$

(5) $2X^2 + 4 = 12$

$\because 2X^2 = 12 - 4$ $\therefore 2X^2 = 8$

$\therefore X^2 = \frac{8}{2}$ $\therefore X^2 = 4$

$\therefore X = \pm\sqrt{4} = \pm 2$

$S.S. = \{2 ، -2\}$

(6) $3X^2 - 1 = 2X^2 + 48$

$\because 3X^2 - 2X^2 = 48 + 1$ $\therefore X^2 = 49$

$\therefore X = \pm\sqrt{49} = \pm 7$

$S.S. = \{7 ، -7\}$

Exercise

Find the solution set for each of the following equations in Z

(1) $X^2 = 64$

(2) $X^2 + 3 = 103$

(3) $X^2 - 5 = 59$

(4) $2X^2 - 5 = 3$

(5) $3X^2 + 4 = 7$

(6) $5X^2 + 2 = 3X^2 + 10$

Home work on the square root

Q1 Choose the correct answer from the given answers

1 What is the value of $\sqrt{(-5)^2}$?

(a) 25 (b) ± 5 (c) 5 (d) -5

2 If $\sqrt{X} = 5$: what is the value of x?

(a) ± 25 (b) 25 (c) 20 (d) 10

3 What is the value of the multiplicative inverse of the number $\sqrt{\frac{9}{25}}$ In the simplest form؟

(a) $\frac{5}{3}$ (b) $-\frac{5}{3}$ (c) $\frac{3}{5}$ (d) $-\frac{3}{5}$

4 If b , a :are the square roots of the number c So what's the value of $a+b$

(a) 2a (b) 2b (c) 1 (d) 0

5 If : $\sqrt{9+16} = 3 + X$ what is the value of x ?

(a) 1 (b) 2 (c) 4 (d) 7

6 What is the value of $\sqrt{10^2 - 6^2}$?

(a) 4 (b) 8 (c) ± 4 (d) ± 8

7 The additive inverse of the number $-\sqrt{\frac{25}{9}}$

(a) $\frac{25}{9}$ (b) $\frac{5}{3}$ (c) $\frac{3}{5}$ (d) $-\frac{5}{3}$

8 $-\sqrt{\frac{49a^4}{25b^8}} = ...?$

(a) $\frac{7a^4}{5b^8}$ (b) $\frac{49a^2}{25b^4}$ (c) $\frac{7a^2}{5b^4}$ (d) $\pm\frac{7a^2}{5b^4}$

9 If $X^2 = 4$: what is the value of x ?

(a) 2 (b) ± 2 (c) -2 (d) 16

10 What is the value of $\sqrt{2\frac{1}{4}}$

(a) $1\frac{1}{2}$ (b) $\frac{9}{4}$ (c) $\frac{3}{2}$ (d) $\pm\frac{3}{2}$

Q2 Find the solution set of each of the following equations in Z

1	$X^2 + 3 = 78$	2	$2X^2 = 8$	3	$X^2 = 121$
4	$3X^2 + 13 = 16$	5	$5X^2 - 1 = 19$	6	$X^2 - 2 = 167$
7	$3X^2 + (X^2 + 1) = 101$	8	$3X^2 - 2 = X^2 + 70$	9	$5X^2 + 1 = 4X^2 + 26$

Second — Cube Root of a Whole Cube Number

Notice that

- Cubes of integers $(\pm1)^3$ ، $(\pm2)^3$ ، $(\pm3)^3$ ، $(\pm4)^3$ ، $(\pm5)^3$ ، .... equals ±1 ، ±8 ، ±27 ، ±64 ، ±125 ، ...... Called whole cubes

- Cube Root of a Perfect Cube Number (a) is the number whose cube is equal to (a)
- The cube root of a whole cube has the same sign as the number
- For example : the number 27 its cube root is 3 because ، $(3)^3 = 27$:the number -125 its cube root is -5 because $(-5)^3 = -125$:

Notice that

- The symbol "$\sqrt[3]{\ }$" Expresses the cube root of a number , for example : the cube root for the number 64 is : 4 $\sqrt[3]{64} = 4$

Note that

(1) $\sqrt[3]{0} = 0$ (2) $\sqrt[3]{-a} = -\sqrt[3]{a}$

(3) $\sqrt[3]{a^{3n}} = a^n$ ، n integer for example $\sqrt[3]{X^{12}} = X^4$ ، $\sqrt[3]{X^3} = X$

125	
5	125
5	25
5	5
	1

(1) $\sqrt[3]{125}$

$125 = 5 \times 5 \times 5$

$\sqrt[3]{125} = 5$

2	216
2	108
2	54
3	27
3	9
3	3
	1

(2) $\sqrt[3]{216}$

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$\sqrt[3]{216} = 2 \times 3 = 6$

(3) $\sqrt[3]{(-7)^3}$	(4) $\sqrt[3]{3\frac{3}{8}}$	(5) $\sqrt[3]{0.125}$	(6) $\sqrt[3]{-\frac{125}{216}}$
-7	$\sqrt[3]{\frac{27}{8}} = \frac{3}{2}$	$\sqrt[3]{\frac{125}{1000}} = \frac{5}{10} = \frac{1}{2} = 0.5$	$-\frac{5}{6}$

$(7)\sqrt[3]{(8)^2}$	$(8)\sqrt[3]{-\left(\frac{2x}{3}\right)^3}$	$(9)\sqrt[3]{\frac{125a^3b^6}{216c^9}}$	$(10)\sqrt[3]{\frac{8x^3}{27y^6}}$
$\sqrt[3]{64}=4$	$-\frac{2x}{3}$	$\frac{5ab^2}{6c^3}$	$\frac{2X}{3Y^2}$

Exercise

.Find each of the following in simplest form

$(1)\sqrt[3]{(8)^3}$	$(2)-\sqrt[3]{0.027}$	$(3)\sqrt[3]{343}$
$(4)\sqrt[3]{\frac{8x^6}{27y^3}}$	$(5)\sqrt[3]{\left(\frac{1}{8}\right)^2}$	$(6)-\sqrt[3]{\frac{125}{64}}$
$(7)\sqrt[3]{15\frac{5}{8}}$	$(8)\sqrt[3]{-8}-\sqrt{4}$	$(9)\sqrt[3]{-125}+\sqrt{25}$

Example 2

Simplify each of the following

$(1)\sqrt{\frac{81}{49}}+\left(\frac{3}{4}\right)^0+\sqrt[3]{\frac{125}{343}}$	$(2)\sqrt[3]{\frac{729}{64}}\times\sqrt{\frac{64}{9}}+\times\left(\frac{-8}{3}\right)^0$	$(3)\sqrt[3]{-\frac{8}{27}}+\sqrt{\frac{4}{9}}+\sqrt[3]{125}$
$\frac{9}{7}+1+\frac{5}{7}=\frac{14}{7}+1=2+1=3$	$\frac{9}{4}\times\frac{8}{3}\times1=6$	$\left(-\frac{2}{3}\right)+\frac{2}{3}+5=5$

Exercise

Simplify each of the following

$(1)\left(\frac{2}{3}\right)^2\times\sqrt[3]{-\frac{27}{8}}\times\sqrt{\frac{4}{9}}$	$(2)\sqrt[3]{-\frac{27}{64}}\times\left(-\frac{3}{7}\right)^0\times\sqrt{\left(\frac{4}{3}\right)^2}$	$(3)\sqrt[3]{\frac{125}{216}}+\left(\frac{3}{7}\right)^0+\sqrt{\frac{1}{36}}$

Solving equations using cube root

If $X^3 = a$: then $X = \sqrt[3]{a}$:

for example : if $X^3 = 125$ then : $X = \sqrt[3]{125} = 5$

Example

.Find the solution set for each of the following equations in Z

$(1) X^3 = 216$	$(2) X^3 + 2 = 127$	$(3) X^3 + 10 = 2$
$X = \sqrt[3]{216} = 6$ $S.S. = \{6\}$	$\because X^3 = 127 - 2$ $\therefore X^3 = 125$ $\therefore X = \sqrt[3]{125} = 5$ $S.S. = \{5\}$	$\because X^3 = 2 - 10$ $\therefore X^3 = -8$ $\therefore X = \sqrt[3]{-8} = -2$ $S.S. = \{-2\}$
$(4) 3X^3 - 1 = 23$	$(5) 2X^3 + 4 = 132$	$(6) 3X^3 + 1 = 2X^3 - 26$
$\because 3X^3 = 23 + 1$ $\therefore 3X^3 = 24$ $\therefore X^3 = \frac{24}{3}$ $\therefore X^3 = 8$ $\therefore X = \sqrt[3]{8} = 2$ $S.S. = \{2\}$	$\because 2X^3 = 132 - 4$ $\therefore 2X^3 = 128$ $\therefore X^3 = \frac{128}{2}$ $\therefore X^3 = 64$ $\therefore X = \sqrt[3]{64} = 4$ $S.S. = \{4\}$	$\because 3X^3 - 2X^3 = -1 - 26$ $\therefore X^3 = -27$ $\therefore X = \sqrt[3]{-27} = -3$ $S.S. = \{-3\}$
$(7) (X-2)^3 = 125$	$(8) (X+3)^3 = -64$	$(9) (X-1)^3 + 2 = 10$
$X - 2 = \sqrt[3]{125}$ $\therefore X - 2 = 5$ $X = 5 + 2 = 7$ $S.S. = \{7\}$	$X + 3 = \sqrt[3]{-64}$ $\therefore X + 3 = -4$ $X = -4 - 3 = -7$ $S.S. = \{-7\}$	$\because (X-1)^3 = 10 - 2$ $\therefore (X-1)^3 = 8$ $\therefore X - 1 = \sqrt[3]{8} = 2$ $\therefore X = 2 + 1 = 3$ $S.S. = \{3\}$

Exercise

.Find the solution set for each of the following equations in Z

$(1) X^3 = 343$	$(2) X^3 + 2 = -25$	$(3) X^3 - 6 = -14$
$(4) 5X^3 + 3 = 8$	$(5) -2X^3 + 4 = 58$	$(6) 3X^3 + 2 = X^3 + 18$

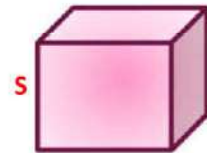
Exercise

$(7)(X+3)^3+5=130$	$(8)(X-6)^3=-343$	$(9)(X+5)^3=216$

Remember

⊙ If the cube edge length (S) then :

(1) Lateral area $= 4s^2$ (2) Total area $= 6s^2$ (3) Volume $= s^3$

Example 4

Answer the following questions

(1) A cube has a volume of 343 cubic units. What is the length of its edges?

Let the cube edge length is (S) , then its volume S^3

$\therefore S^3 = 343$ $\therefore S = \sqrt[3]{343} = 7$

(2) A cube has a volume of 125 cm3. What is its lateral and total (surface) area?

Let the cube edge length is (S) , then its volume S^3

$\therefore S^3 = 125$ $\therefore S = \sqrt[3]{125} = 5$

Then, the length of the cube's edge is 5 cm.

$\therefore$ The lateral area $= 4S^2 = 4\times(5)^2 = 100 \text{ cm}^2$

$\therefore$ The total area $= 6S^2 = 6\times(5)^2 = 150 \text{ cm}^2$

Exercise

Answer the following questions

(1) A cube has a volume of 512 cubic units. What is the length of its edges ?

(2)A cube has a volume of 216 cm3. What is its lateral and total (surface) area?

Home work — On the cube root

Q1 Choose the correct answer from the given answers

1 If $a = 5^3$ what is the value of $\sqrt[3]{a}$?

3 (a) 5 (b) 25 (c) 125 (d)

2 If $X^3 = -27$ what is the value of x ?

−3 (a) 3 (b) ±3 (c) −9 (d)

3 what is the value of $\sqrt{4} - \sqrt[3]{-8}$?

12 (a) 4 (b) −4 (c) 0 (d)

4 إذا كان: $\sqrt[3]{b} = -8$ فما قيمة b ؟

−512 (d) 64 (c) −2 (b) 2 (a)

5 I
f

−5 (a) 5 (b) 125 (c) −125 (d)

6 What is the value of $\sqrt[3]{\sqrt{64}}$?

2 (a) 4 (b) 8 (c) 64 (d)

7 what is the value of $\sqrt[3]{(-8)^2}$?

2 (a) −2 (b) 4 (c) −4 (d)

8 If : $X = \sqrt{\frac{1}{9}}$ what is the value of X^3 ?

$\frac{1}{3}$ (a) $\frac{1}{9}$ (b) $\frac{1}{27}$ (c) $\frac{1}{81}$ (d)

9 $\sqrt[3]{X^6} = \sqrt{........}$

X^3 (a) X (c) X^2 (b) X^4 (d)

10 what is the value of $\sqrt[3]{3\frac{3}{8}} + \sqrt{0.25}$?

$\frac{3}{2}$ (a) $\frac{1}{2}$ (b) 2 (c) −2 (d)

11 If : $X^3 = 46$ what is the value of $\sqrt{X}$?

−4 (a) 2 (b) 4 (c) −2 (d)

12 what is the value of $\sqrt{(-2)^2} + \sqrt[3]{(-2)^3}$?

−4 (a) 8 (b) 4 (c) 0 (d)

Q2 Find the solution set of each of the following equations in Z

1	$X^3 + 3 = 128$	2	$3X^3 = -24$	3	$X^3 = 216$
4	$(X+7)^3 = 125$	5	$5X^3 - 1 = -41$	6	$X^3 - 2 = 341$
7	$(X+4)^3 + 5 = 32$	8	$3X^3 - 1 = 2X^3 + 124$	9	$(X-1)^3 = -8$

Q3	Find the value of each of the following				
1	$\sqrt[3]{64 \times 2^3}$	2	$\sqrt[3]{-1}$	3	$\sqrt[3]{512}$
4	$\sqrt[3]{\frac{8X^3}{125Y^6}}$	5	$\sqrt{4} - \sqrt[3]{-8}$	6	$\sqrt[3]{343 - 7^3}$
7	$\sqrt[3]{\frac{-8}{27}} \times \sqrt{\frac{9}{4}}$	8	$\sqrt[3]{(-8)^3} + \sqrt{64}$	9	$\sqrt[3]{0.125} + \sqrt{12\frac{1}{4}}$

Q4	Simplify each of the following				
1	$\sqrt[3]{\frac{125}{27}} \times \sqrt{\frac{81}{25}} \times \left(\frac{5}{3}\right)^0$	2	$\left(\frac{2}{3}\right)^2 + \sqrt{\frac{16}{9}} + \sqrt[3]{\frac{8}{27}}$	3	$\sqrt{\frac{25}{9}} + \sqrt[3]{\frac{-125}{27}} + \left(\frac{9}{25}\right)^0$
4	$\sqrt[3]{3\frac{3}{8} \times \frac{2}{3} \times \left(\frac{3}{2}\right)^0}$	5	$\left(\frac{1}{2}\right)^2 \times \sqrt{\frac{4}{9}} \times \sqrt[3]{\frac{27}{16}}$	6	$\sqrt{\frac{81}{49}} + \left(\frac{9}{7}\right)^0 + \sqrt[3]{\frac{125}{343}}$

Q5	Answer the following questions
1	A cube has a volume of 64 cubic units find its edge length
2	A cube has a volume of 27 cm3, what is its lateral and total area (surface) ?

Lesson 4 Inequalities

The Inequality

⊙ The inequality consists of two mathematical expressions between them one of the signs of variation $(\geq ، \geq ، > ، <)$

- $X<3$ " and reads X less than 3 "
- $X>6$ " and reads X greater than 6 "
- $X\leq -4$ " and reads X less than or equal to -4 "
- $X\geq 1$ " and reads X greater than or equal to 1 "

First degree inequality in one variable

⊙ Inequalities like $3X-1<5$, $2X\geq 8$, $2X-1\leq X+3$ They are called first-degree inequalities in one variable because they contain only one variable raised to the power of one .

Solving an Inequality Using inequality Properties

⊙ When solving first-order inequality in one variable, we use the same method used when solving first-order equations in one variable, taking into account the properties of inequalities

Properties of inequalities

If A, B, and C are three numbers, these numbers have the following properties:

Add property

If : $A>B$ Then $A+C>B+C$

For example $7>4$ Then $7+2>4+2$ i.e. $9>6$

⊙ That is, when adding the same number to both sides of the inequality, it remains true

Subtraction property

If : $A>B$ Then: $A-C>B-C$

For example $7>4$ Then: $7-2>4-2$ i.e. $5>2$

⊙ That is, when the same number is subtracted or eliminated to both sides of the inequality, it remains true

Multiplication property

If: $A>B$ Then: $A\times C>B\times C$ " $C>0$ "positive"

If : $7>4$ Then $7\times 2>4\times 2$ i.e. $14>8$

⊙ That is, when the same positive number is multiplied by both sides of the inequality, it remains true

If : $A>B$ Then: $A\times C<B\times C$ " $C<0$ " negative"

if : $7>4$ Then: $7\times(-2)>4\times(-2)$ i.e. $-14<-8$

⊙ That is, when the same negative number is multiplied by both sides of the inequality, the direction of the variance changes until the

Divisibility property

If : $A>B$ Then: $\frac{A}{C}>\frac{B}{C}$ "$C>0$" "positive" If : $6>4$ then : $\frac{6}{2}>\frac{4}{2}$ i.e. $3>2$ ⊙ That is, when dividing the two sides of the inequality by a positive number, it remains true	If: $A>B$ Then: $\frac{A}{C}<\frac{B}{C}$ "$C<0$" : "negative" If : $6>4$ Then : $\frac{6}{-2}>\frac{4}{-2}$ i.e. $-3<-2$ ⊙ That is, when dividing the two sides of the inequality by a negative number, the direction of the variance changes so that the inequality remains true

Writing an inequality

Example 1

Express each of the following situations with appropriate inequality

(1) A sports stadium can accommodate a maximum of 5,000 spectators	(2) You must be more than 160 cm tall to participate in one of the basketball teams	(3) You must be at least 16 years old to obtain a national ID card
Let the number of people X $\therefore X \le 5000$	Let the length be X $\therefore X>160$	Let age be X $\therefore X \ge 16$

(4) If 4 subtracts from three times a number, the result is less than or equal to 8	(5) If 3 is added to twice the number, the result is greater than 11
Let the number X $\therefore 3X-4 \le 8$	Let the number X $\therefore 2X+3>11$

Exercise

Express each of the following situations with appropriate inequality

(1) A classroom in a school can accommodate a maximum of 40 students	(2) You must weigh more than 70 kg to join the boxing team	(3) You must be at least 18 years old to obtain a driver's license

(4) If subtract 2 from twice a number, the result is less than 7	(5) If 5 is added to four times a number, the result is greater than or equal to 18

Example 2

Find the solution set of the following inequalities in (1) $\mathbb{N}$ (2) $\mathbb{Z}$ (3) $\mathbb{Q}$

(1) $2-3X<-7$

By adding (-2) to both sides $\therefore -2+2-3X<-7-2$

$\therefore -3X<-9$

Dividing both sides by (3 -) $\therefore \frac{-3X}{-3}>\frac{-9}{-3}$

$\therefore X>3$

(1) S.S. in $\mathbb{N}=\{4, 5, 6, 7, ...\}$

(2) S.S. in $\mathbb{Z}=\{4, 5, 6, 7, ...\}$

(3) S.S. in $\mathbb{Q}=\{a : a \in \mathbb{Q}, a>3\}$

(2) $2x-1\le 7$

By adding (1) to both sides $\therefore 2X-1+1\le 7+1$

$\therefore 2X\le 8$

By dividing both sides by (2) $\therefore \frac{2X}{2}\le\frac{8}{2}$

$\therefore X\le 4$

(1) S.S. In $\mathbb{N}=\{3, 2, 1, 0\}$

(2) S.S. In $\mathbb{Z}=\{3, 2, 1, 0, -1, -2, ...\}$

(3) S.S. in $\mathbb{Q}=\{a : a \in \mathbb{Q}, a\le 3\}$

(3) $5x\ge 10$

Dividing both sides by (5) $\therefore \frac{5X}{5}\ge\frac{10}{5}$

$\therefore X\ge 2$

(1) S.S. in $\mathbb{N}=\{2, 3, 4, 5, ...\}$

(2) S.S.in $\mathbb{Z}=\{2, 3, 4, 5, ...\}$

(3) S.S. in $\mathbb{Q}=\{a : a \in \mathbb{Q}, a\ge 2\}$

(4) $X+3>-1$

Subtract (3) from both sides $\therefore X+3-3>-1-3$

$\therefore X>-4$

(1) S.S. in $\mathbb{N}=\{0, 1, 2, 3, ...\}$

(2) S.S. in $\mathbb{Z}=\{-3, -2, -1, 0, 1, ...\}$

(3) S.S. in $\mathbb{Q}=\{a : a \in \mathbb{Q}, a>-4\}$

Exercise

Find the solution set of the following inequalities in (1) $\mathbb{N}$ (2) $\mathbb{Z}$ (3) $\mathbb{Q}$

(1) $3-2x>-5$

(2) $5x-3\le 17$

Example 3

Find in Z the solution set of each of the following inequalities

$(1)\ 2(5X-1)>8$

By dividing both sides by(2) $\therefore \frac{2(5X-1)}{2}>\frac{8}{2}$

$\therefore 5X-1>4$

By adding (1) to both parties

$\therefore 5X-1+1>4+1$ $\therefore 5X>5$

Dividing both sides by (5)

$\therefore \frac{5X}{5}>\frac{5}{5} \Rightarrow \therefore X>1$

S.S. in $\mathbb{Z}=\{2, 3, 4, 5, ...\}$

$(2)\ 2-3X<11$

By subtracting (2) from both sides of the inequality

$\therefore 2-2-3X<11-2$

$\therefore -3X<9$

By dividing both sides by(3-) $\therefore \frac{-3X}{-3}>\frac{9}{-3}$

$\therefore X>-3$

S.S. in $\mathbb{R}=\{-2, -1, 0, 1, 2, ...\}$

$(3)\ X+1>4X-2$

Grouping similar terms $\therefore X-4X>-2-1$

$\therefore -3X>-3$

By dividing both sides by (3-) $\therefore \frac{-3X}{-3}<\frac{-3}{-3}$

$\therefore X<1$

S.S. in $\mathbb{Z}=\{0, -1, -2 ....\}$

$(4)\ 3X-5\le X+1$

Grouping similar terms $\therefore 3X-X\le 1+5$

$\therefore 2X\le 6$

Dividing both sides by (2) $\therefore \frac{2X}{2}\le\frac{6}{2}$

$\therefore X\le 3$

S.S. $=\{3, 2, 1, 0, -1, -2 ....\}$

Exercise

Find in Z the solution set of each of the following inequalities

$(1)\ 4-3X<10$

$(2)\ 3(2X-1)>-9$

Solving first-degree inequality in one variable — Homework

Q1 Choose the correct answer from the given answers

1 Which of the following inequalities expresses the following situation:

"It takes at least two hours to complete the homework"

(a) $X < 2$ (b) $X \le 2$

(c) $X > 2$ (d) $X \ge 2$

2 What inequality expresses that the temperature X is less than 40° ؟

(a) $X < 40^\circ$ (b) $X > 40^\circ$

(c) $X \le 40^\circ$ (d) $X \ge 40^\circ$

3 If $X - 1 > 4$ Which of the following could be the value of X ?

(a) 3 (c) 5 (b) 4 (d) 7

4 What inequality shows that double X is less than 5

(a) $X + 2 < 5$? (b) $X - 2 < 5$

(c) $2X < 5$ (d) $2X > 5$

5 Which of the following inequalities has one of its solutions in Q is $X = -4$ ؟

(a) $X - 2 \ge -4$ (b) $2X > -8$

(c) $X + 2 > -3$ (d) $-X > 4$

6 Which of the following inequalities has one of its solutions in Z is $X = -7$?

(a) $X > -7$ (b) $X < -7$

(c) $X > -6$ (d) $-X \ge -7$

7 If $1 - X > 4$ then ....

(a) $X < 3$ (b) $X > -3$

(c) $X > 3$ (d) $X > 3$

8 If $-5X < 15$ then ....

(a) $X < 3$ (b) $X > -3$

(c) $X > 3$ (d) $X > 3$

9 If: $X \in \mathbb{Z}$ Which of the following is one of the solutions to the inequality $1 - 2X < 3$

(a) 0 (b) -1

(c) -2 (d) -4

10 The S.S. of the inequality $-X > -1$ in N is

(a) $\{0, 1, 2, ...\}$ (b) $\{0\}$

(c) $\{-1, -2, -3, ...\}$ (d) $\{0, -1, -2, ...\}$

11 If : $X \in \mathbb{N}$ Which of the following is one of the solutions to the inequality $2X < 2$

(a) 0 (b) -1

(c) 1 (d) -4

12 If : $X \in \mathbb{Z}$ Which of the following is one of the solutions to the inequality $X + 1 > 2$

(a) 0 (b) 1

(c) -1 (d) 2

Q2 Find the solution set of each of the following invariants in N

1. $3X \geq 6$
2. $2X < 8$
3. $X - 1 > 3$
4. $\frac{1}{2}X + 1 < 3$
5. $2 + 3X \leq 5$
6. $2X - 1 < 5$

Q3 Find the solution set of each of the following inequalities in Z

1. $(2 - 3X) < 4$
2. $2(2X + 3) > 14$
3. $2 - 3X \geq 11$
4. $3X - 2 \geq X + 6$
5. $5X - 1 > 4X + 2$
6. $3X + 7 < 7X + 3$

Q4 Find the solution set of each of the following inequalities in Q

1. $X - 2 \leq 3X + 7$
2. $5 - 3X \geq 14$
3. $5X + 2 > -3$
4. $5(X + 2) \leq 5$
5. $X - 1 \geq 2X - 3$
6. $3(X - 7) \geq 7(X - 3)$

Lesson 5 Multiplying algebraic term by algebraic term or an algebraic expression

Multiplying algebraic term by algebraic term

⊙ When multiplying an algebraic term by another algebraic term, we multiply the coefficients and add the exponents of the variables that have the same base $(\geq ، \geq ، > ، <)$

⊙ $aX^m \times bX^n = a\times b X^{m+n}$

⊙ For example $2X^4 \times 3X^5 = (2\times 3)X^{4+5} = 6X^9$

Example 1

Find the result of each of the following

$(1)(2a^2b)(4ab^3)$	$(2)(-5x^3)(-2x^2)$	$(3)(2a^2)(5a^4)$
$(2\times 4)\times a^{2+1}\times b^{1+3} = 8a^3b^4$	$(-5\times -2)\times(x^{3+2}) = 10x^5$	$=(2\times 5)\times(a^{2+4}) = 10a^6$
$(4)(-r^3s^2t)(2r^2s^3)$	$(5)(2c^3d^2e)(c^2de^3)$	$(6)(-2x^3y)(4y^5)$
$(-1\times 2)\times r^{3+2}\times s^{2+3}\times t = -2r^5s^5t$	$(2\times 1)\times c^{3+2}\times d^{2+1}\times e^{1+3} = 2c^5d^3e^4$	$=(-2\times 4)\times x^3\times y^{1+5} = -8x^3y^6$

Exercise

Find the result of each of the following

$(1)(5r^2s^2t)(-rt^4)$	$(2)(-4x^2y)(-6xy^3)$	$(3)(-3a^2)(4a^5)$
$(4)(8x^3y)(-2x^2y^2z)$	$(5)(-3a^3b^2)(-2ab^4)$	$(6)(2a)(-a^2)(3a^3)$

Multiplying algebraic term by an algebraic expression with two or more terms

- When multiplying an algebraic term by an algebraic expression with two or more terms, we use the distribution property:

$a(b+c)=ab+ac$ $a(b-c)=ab-ac$

$x(x-3)=(4x)(x)-(4x)(3)=4x^2-12x$

$3x(x+2)=(3x)(x)+(3x)(2)=3x^2+6x$

Example 2

Find the result of each of the following

$(1)\ 2a(3a-5)$	$(2)\ 5(4+3x)$	$(3)\ 5x(2x^2-1)$
$(2a)(3a)-(2a)(5)=6a^2-10a$	$(5)(4)+5(3x)=20+15x$	$(5x)(2x^2)-(5x)(1)=10x^3-5x$
$(4)\ 2a(3a^2b+5)$	$(5)\ 2ab(3a+4b)$	$(6)\ -3x(x-2)$
$(2a)(3a^2b)+(2a)(5)=6a^3b+10a$	$(2ab)(3a)+(2ab)(4b)=6a^2b+8ab^2$	$(-3x)(2)=-3x^2-(-6x)=-3x^2+6x$

$(7)\ 2x^2(4x^2-5x-7)$	$(8)\ 2ab(3a^2+4b-5)$
$(2x^2)(4x^2)-(2x^2)(5x)-(2x^2)(7)=8x^4-10x^3-14x^2$	$(2ab)(3a^2)+(2ab)(4b)-(2ab)(5)=6a^3b+2ab^2-10ab$

Example 3

Simplify each of the following for the simplest form

$(1)\ 3(x+2y)+4x$

$(3)(x)+(3)(2y)+4x=3x+6y+4x$
$=(3x+4x)+6y=7x+6y$

$(2)\ 2x(x+3)+3(x-1)$

$(2x)(x)+(2x)(3)+(3)(x)-(3)(1)=$
$=2x^2+6x+3x-3=2x^2+9x-3$

$(3)\ 2x(3x-5)+5x(x-2)$

$(2x)(3x)-(2x)(5)+(5x)(x)-(5x)(2)=$
$=6x^2-10x+5x^2-10x=11x^2$

$(4)\ 2(5x^2+4x-1)-10x^2$

$(2)(5x^2)+(2)(4x)-(2)(1)-10x^2$
$=10x^2+8x-2-10x^2=8x-2$

Exercise

Simplify each of the following for the simplest form

(1) $2x(x+3)+4x^2$

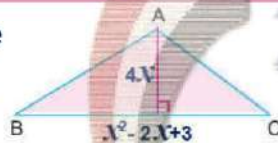
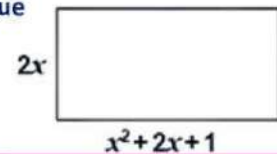
(2) $-3x(5-2x)-6x^2$

(3) $2x(x+3)+3x(x-1)$

(4) $3x(x^2-2x+1)+4x^2$

Example 4

Answer the following questions

(1) Find the area of the triangle in terms of x, then calculate the numerical value for the area when $x=2$

(1) Find the Area of the Rectangle in terms of x, then calculate the numerical value for the area when $x=2$

Area of the triangle $=\frac{1}{2}$ Length of the base $\times$ Height

$=\frac{1}{2}(4x)(x^2-2x+3)=(2x)(x^2-2x+3)$

$=2x^3-4x^2+6x$

when $x=2$

The area $= 2(2)^3-4(2)^2+6(2)=16-16+12=12$

Area of rectangle $=$ Length $\times$ Width

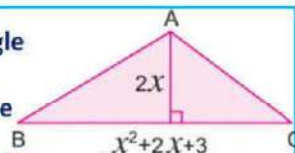
$=2x(x^2+2x+1)=2x^3+4x^2+2x$

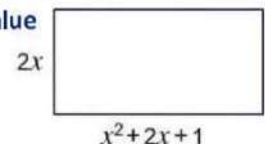
when $x=2$

the area $=2(2)^3+4(2)^2+2(2)=16+16+4=36$

Exercise

Answer the following questions

(2) Find the area of the triangle in terms of x, then calculate the numerical value for the area when $x=1$

A
2x
B
X^2+2X+3
C

(1) Find the Area of the Rectangle in terms of x, then calculate the numerical value for the area when $x=3$

Exercise

Example 5

simplify each of the following to the simplest form

$3x(x+2)+2x(x-1)$ then find the numerical value of the result at $x=2$

$=3x^2+6x+2x^2-2x$

$=5x^2+4x$

when $x=2$

$5(2)^2+4(2)=20+8=28$

Exercise

simplify each of the following to the simplest form

$5x(x-1)+x(3x+2)$ then find the numerical value of the result at $x=3$

Example 6

Find the solution set of the equation

$2x(x-1)+x(x+2)=12$ in Z

$\because 2x^2-2x+x^2+2x=12$

$\therefore 3x^2=12 \Rightarrow x^2=\frac{12}{3}$

$\therefore x^2=4 \Rightarrow x=\pm 2$

$\therefore S.S.=\{2,-2\}$

Exercise

Find the solution set of the equation

$3x(x+2)+x(x-6)=36$ in Z

home work — Multiplying algebraic term by algebraic term or algebraic expression

Q1 Choose the correct answer from the given answers

1. $(-2x^2)(4x^3) = ...$?
(a) $-12x^5$ (b) $12x$
(c) $-12x^6$ (d) $12x^5$

2. $(2x)(3x) = ...$?
(a) $5x$ (b) $6x$
(c) $5x^2$ (d) $6x^2$

3. $x(x-1) + x = ...$
(a) $x(2x-1)$ (b) $2x^2$
(c) x^2 (d) $x^2 - x$

4. $2(x+3) = ...$?
(a) $5x$ (b) $6x$
(c) $5x^2$ (d) $6x^2$

5. Which of the following inequalities has one of its solutions in Q is $X = -4$?
(a) $X - 2 \geq -4$ (b) $2X > -8$
(c) $X + 2 > -3$ (d) $-X > 4$

6. $-3ab \times -3ab = ...$
(a) $-6ab$ (c) $9a^2b^2$
(d) $-9a^2b^2$ (b) $6ab$

7. If : $3a^m \times 2a^3 = 6a^3$ what is the value of a ?
(a) 6 (b) 2
(c) 1 (d) 3

8. If : $-2x^3 \times x = ax^n$ So what's the value of $a + n$?
(a) -3 (b) -2
(c) 2 (d) 3

9. $-2x^2y \times -3y^2x = ...$
(a) $-6x^4y^2$ (b) $6x^4y^2$
(c) $6x^3y^3$ (d) $-6x^3y^3$

10. a rectangle ، $(x+1)$cm ، $(2x)$cm then its area .....
(a) $2x+1$ (b) $3x+1$
(c) $2x^2+1$ (d) $2x^2+2x$

Q2 Find the result of each of the following

1. $(-4m)(-6m^5)$
2. $(a^2b^3)(4a^4b^3)$
3. $(-5x^2y^2)(3xy)$
4. $\left(\frac{1}{2}x^2y^4\right)(2xy^3)$
5. $3x \times 5xy \times 2y$
6. $(7p^3s^2r)(-4p^2r)$
7. $(-2x^2)(3y^2)(5xz^3)$
8. $\left(\frac{1}{3}m^4n^2\right)\left(-\frac{1}{2}m\right)(6n)$
9. $(2ab^2)(-a^2b)(-3b^3)$

Q3 Find the result of each of the following

3. $4ab(2a+3b)$
2. $-5x(2+3x)$
3. $-4a(3a-2)$
4. $5a^2b(2ab^2-3a^2b)$
5. $2x^2y(3x-4y+1)$
6. $2x(3x^2-5)$
7. $2xyz(5x-3y+2z)$
8. $-ab(2a^2b-3ab^3+2ab)$
9. $2x(4x^2-xy+5)$

Q4 Simplify each of the following for the simplest form

3. $5y-4(x-y+1)$
2. $2x(x+3)+3(x-1)$
3. $3(x+2y)+4x$
4. $3x^2(x^2+2x+3)-3x(x^3-x^2)$
5. $2a(3a-1)+a(3-6a)$
6. $3x(2x-y)+3y(x-2y)$

Q5 Simplify each of the following for the simplest form

1. $2a(3a+b)-3b(a+b)$ then find the numerical value of the result at $a=b=1$
2. $5x(x+1)-2x(3x+2)$ then find the numerical value of the result at $x=2$
3. $2a(3a-1)+3a(a+1)$ then find the numerical value of the result at $a=1$

Q6 Solve each of the following equations in Z

1. $x(x-2)+2(x-2)=0$
2. $2x(x+1)-x(x+2)=9$
3. $x(2x+3)-3x=50$

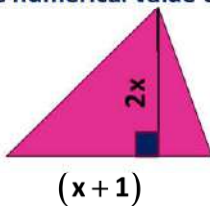
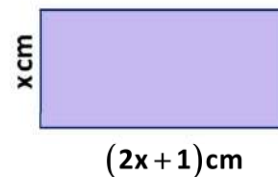
Q7 Answer the following questions

1. Find the area of the triangle in terms of x, then calculate the numerical value of the area at $x=4$

$(x+1)$

2. Find the Area of the Rectangle in terms of x, then calculate the numerical value of the area at $x=5$

x cm

$(2x+1)$cm

Lesson 6 Multiplying algebraic expressions

Multiplying an algebraic binomial expression by another binomial

- When multiplying a binomial algebraic expression by another binomial algebraic expression, we multiply each term of the first expression by the term of the second expression using the distribution property.

$$(x+3)(y+2)=x(y+2)+3(y+2)$$
$$=xy+2x+3y+6$$

$$(x+a)(y+b)=x(y+b)+a(y+b)$$
$$=xy+xb+ay+ab$$

Notice that

- Multiplying an algebraic binomial by another binomial produces an algebraic expression of 4 terms
- If there are similar terms, they are added so that the expression is in the simplest form

$$(x+3)(x+4)=x(x+4)+3(x+4)$$
$$=x^2+4x+3x+12$$
$$=x^2+7x+12$$

$$(x+a)(x+b)=x(x+b)+a(x+b)$$
$$=x^2+xb+ax+ab$$
$$=x^2+(b+a)x+ab$$

Direct multiplication

- You can multiply directly as follows

$$=x^2+7x+12$$

Vertical method

- The vertical method can be used to find the product as follows

$$x^2+3x$$
$$+4x+12$$
$$x^2+7x+12$$

Example 1

Find the product of each of the following

(1) $(x+2)(x+3)$	(2) $(a-5)(a-3)$	(3) $(y-5)(y+7)$
$x^2+(2x+3x)+6=x^2+5x+6$	$a^2+(-3a-5a)+15=a^2-8a+15$	$y^2+(-5y+7y)-35=y^2+2y-35$
(4) $(2x-1)(3x+2)$	(5) $(2a+5)(a+1)$	(6) $(x-5)(x+1)$
$6x^2+(4x-3x)-2=6x^2+x-2$	$2a^2+(2a+5a)+5=2a^2+7a+5$	$x^2+(-5x+x)-5=x^2-4x-5$

Example 1

Find the product of each of the following

(7) $(3m+2n)(2m-3n)$	(8) $(a-6)(3a+1)$	(9) $(3a+b)(a+2b)$
$6m^2+(4m-9m)-6n^2$ $=6m^2-5n-6n^2$	$3a^2+(a-18a)-6=3a^2-17a-6$	$3a^2+(6ab+ab)+2b^2$ $=3a^2+7ab+2b^2$

Exercise

Find the product of each of the following

(1) $(a+7)(a-2)$	(2) $(y-2)(y-4)$	(3) $(x+5)(x+3)$
(4) $(2y+7)(2y+3)$	(5) $(2x+3)(x+1)$	(6) $(b-8)(b+2)$
(7) $(2a+b)(3a+4b)$	(8) $(2-3x)(5-2x)$	(9) $(2x+y)(x-3y)$

Example 2

Answer the following question

A rectangle its to dimensions are $(x+1)$ ، $(2x+3)$ length unit ، Find its surface area in terms of X ، then find the numerical value of the area at $X=2$

The area of the rectangle =

$(x+1)(2x+3)=2x^2+(2x+3x)+3=2x^2+5x+3$

the numerical value of the area at $X=2$

$2(2)^2+5(2)+3=8+10+3=21$

Exercise

Answer the following question

A rectangle its to dimensions are $(2x+1)$ ، $(x+2)$ length unit ، Find its surface area in terms of X ، then find the numerical value of the area at $X=5$

Special cases

2 square expansion of a binomial algebraic expression

$$(a+b)^2 = (a+b)(a+b)$$
$$= a(a+b) + b(a+b)$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

$$(a-b)^2 = (a-b)(a-b)$$
$$= a(a-b) - b(a-b)$$
$$= a^2 - ab - ab + b^2$$
$$= a^2 - 2ab + b^2$$

For example : $(x+5)^2 = x^2 + 10x + 25$

For example : $(x-6)^2 = x^2 - 12x + 36$

Example 2

Find the expansion of each of the following

$(1)\ (x+2)^2$	$(2)\ (x-3)^2$	$(3)\ (2a+1)^2$
$x^2 + 2\times 2\times x + 2^2 = x^2 + 4x + 4$	$x^2 - 2\times 3\times x + 3^2 = x^2 - 6x + 9$	$(2a)^2 + 2\times 2a\times 1 + 1^2 = 4a^2 + 4a + 1$
$(4)\ (1-6x)^2$	$(5)\ (4+3b)^2$	$(6)\ (3x-5)^2$
$(1)^2 - 2\times 1\times 6x + (6x)^2 = 1 - 12x + 36x^2$	$(4)^2 + 2\times 4\times 3b + (3b)^2 = 16 + 24b + 9b^2$	$(3x)^2 - 2\times 5\times 3x + (5)^2 = 9x^2 - 30x + 25$

Exercise

Find the expansion of each of the following

$(1)\ (x-7)^2$	$(2)\ (9-a)^2$	$(3)\ (3+2a)^2$
$(4)\ (3-4b)^2$	$(5)\ (2ab+3)^2$	$(6)\ (3a-4b)^2$

2 The product of the sum of two terms by the difference between them

$$(a+b)(a-b)=a(a-b)+b(a-b)$$
$$=a^2-ab+ab-b^2$$
$$=a^2-b^2$$

First term square — Second term square

For example :

$$(x-5)(x+5)=x^2-(5)^2=x^2-25$$

Note that : The product of the last two terms is always negative

Example 3

Find in its simplest form each of the following

$(1)(a-3)(a+3)$	$(2)(3x-5)(3x+5)$	$(3)(4+3b)(4-3b)$
$a^2-(3)^2=a^2-9$	$(3x)^2-(5)^2=9x^2-25$	$(4)^2-(3b)^2=16-9b^2$
$(4)(2xy-5)(2xy+5)$	$(5)\left(\frac{1}{3}a-1\right)\left(\frac{1}{3}a+1\right)$	$(6)(5a-2b)(5a+2b)$
$(2xy)^2-(5)^2=4x^2y^2-25$	$\left(\frac{1}{3}a\right)^2-(1)^2=\frac{1}{9}a^2-1$	$(5a)^2-(2b)^2=25a^2-4b^2$

Exercise

Find the expansion of each of the following

$(1)(x+6)(x-6)$	$(2)(a-7)(a+7)$	$(3)(5y+4)(5y-4)$
$(4)\left(3-\frac{1}{2}x\right)\left(3+\frac{1}{2}x\right)$	$(5)(5x-6y)(5x+6y)$	$(6)(5-3b)(5+3b)$

Example 4

Simplify each of the following for the simplest form

$(1)4x^2-(2x-2)(2x+2)$	$(2)(x-5)(x+5)+25$
$4x^2-(4x^2-4)=4x^2-4x^2+4=4$	$x^2-25+25=x^2$

(3) $(2x+3)(2x-3)+9$ then find the numerical value at $x=5$

$(2x)^2-(3)^2+9=4x^2-9+9=4x^2$

when $x=5$ then: $4x^2=4(5)^2=100$

(4) $(x+3)^2-(x-3)(x+3)$

$x^2+6x+9-(x^2-9)=x^2+6x+9-x^2+9$
$=6x+18$

Exercise

Simplify each of the following for the simplest form

(1) $(2a-3)(2a+3)-4a^2$

(2) $(3x-5)(3x+5)+25$

(3) $4x^2-(x+1)(x-1)$ then find the numerical value at $x=2$

(4) $(x-5)(x+5)-(x-5)^2$

Example 5

Answer the following question

The dimensions of a rectangle are $(x-3)$ ، $(x+3)$ unit length Find the surface area in terms of Then find the numerical value of the area when $x=5$

The area of the rectangle :
$(x-3)(x+3)=x^2-9$

the area at $x=5$: $(5)^2-9=25-9=16$

Exercise

Answer the following question

The dimensions of a rectangle are $(2x-1)$ ، $(2x+1)$ unit length Find the surface area in terms of Then find the numerical value of the area when $x=4$

3 Multiplying a binomial algebraic expression by an algebraic expression containing more than two terms

When multiplying a binomial algebraic expression by an algebraic expression consisting of more than two terms, we use the distribution property

Example 4

Find in simplest form the product of each of the following

(1) $(2x-1)(x^2-3x+2)$

$$(2x-1)(x^2-3x+2)=(2x)(x^2-3x+2)-(1)(x^2-3x+2)$$
$$=2x^3-6x^2+4x-x^2+3x-2$$
$$=2x^3-7x^2+7x+6$$

(2) $(x+3)(x^2+2x+1)$

$$(x+3)(x^2+2x+1)=x(x^2+2x+1)+(3)(x^2+2x+1)$$
$$=x^3+2x^2+x+3x^2+6x+6$$
$$=x^3+5x^2+7x+6$$

Exercise

Simplify each of the following

(1) $(3x-2)(x^2+2x+1)$

(2) $(x+5)(x^2-3x+2)$

Example 5

Find the solution set of each of the following equations in Q

(1) $(x+2)(x^2-2x+4)=0$

$\because (x+2)(x^2-2x+4)=0$

$\therefore x(x^2-2x+4)+2(x^2-2x+4)=$

$\therefore x^3-2x^2+4x+2x^2-4x+8$

$\therefore x^3+8=0 \qquad \therefore x^3=-8$

$\therefore x=\sqrt[3]{-8}=-2 \qquad \therefore S.S.=\{-2\}$

(2) $(x+3)(x-3)=16$

$\because (x+3)(x-3)=27 \qquad \therefore x^2-9=27$

$\therefore x^2=27+9 \qquad \therefore x^2=36$

$\therefore x=\pm\sqrt{36} \qquad \therefore x=\pm 6$

$S.S.=\{6\ ,\ -6\}$

Exercise

Find the solution set of each of the following equations in Q

(1) $(x+5)(x^2-5x+25)=0$

(2) $(x+4)(x-4)=9$

Home work — **Multiplying the algebraic expressions**

Q1 Choose the correct answer

1 What is the number of terms of the resulting expression of the product $(X-3)(x+3)$ In the simplest form ?

(a) 1 (b) 2 (c) 3 (d) 4

2 What is the number of terms of the resulting expression of the product $(X-3)(x+4)$ In the simplest form ?

(a) 1 (b) 2 (c) 3 (d) 4

3 If : $(x+5)(x-5)=ax^2+bx+c$ then $b=$ ....

(a) -42 (b) -21 (c) 21 (d) 42

4 I f :

(a) -42 (b) -21 (c) 21 (d) 42

5 If : $(2x-3)^2=ax^2+bx+c$ then $c=$ ....

(a) 4 (b) 9 (c) -9 (d) -12

6 If : $(x-5)(x+2)=ax^2+bx+c$ then $c=$ ....

(a) -42 (b) -21 (c) 21 (d) 42

7 If : $x^2-y^2=6$, $x+y=3$ then $x-y=$ ......

(a) 3 (b) 18 (c) 2 (d) -18

8 If : $x-y=5$, $x+y=15$, then $x^2-y^2=$ ......

(a) 75 (b) 20 (c) 10 (d) 3

9 $(x+3)(x-.....)=x^2-9$

(a) 3 (b) -3 (c) 6 (d) -6

10 If $xy=3$ ، $(x+y)^2=16$ then $x^2+y^2=$ ....

(a) $5\frac{1}{3}$ (b) 10 (c) 13 (d) 48

11 If $xy=6$, $x^2+y^2=9$ then $(x+y)^2=$ ....

(a) 54 (b) 15 (c) 25 (d) 21

12 If : $(x-5)(x+5)=x^2+k$ then $k=$ ....

(a) 25 (b) 10 (c) -25 (d) -10

13 If : $(x+6)(x-6)=x^2+k-36$ then $k=$ ....

(a) 12 (b) -12 (c) -36 (d) 0

14 $(.....-3)^2=4x^2-12x+9$

(a) 2 (b) $2x$ (c) $2x^2$ (d) $4x$

Q2 Multiply each of the following

1	$(y-7)(y+5)$	2	$(x-2)(x+3)$	3	$(a+5)(a+2)$
4	$(3+x)(2x-5)$	5	$(7-x)(2x-1)$	6	$(5-2x)(3-x)$
7	$(1-2ab)(3-ab)$	8	$(2xy-1)(xy+3)$	9	$(2a+3b)(a+5b)$

Q3 Multiply each of the following

1	$(2+x)(x-2)$	2	$(9-2y)(9+2y)$	3	$(x-7)(x+7)$
4	$(2ab+3)(2ab-3)$	5	$(2a-5b)(2a+5b)$	6	$(2x+3)(2x-3)$
7	$(2x^2+3)(2x^2-3)$	8	$(3ab-1)(3ab+1)$	9	$\left(\frac{1}{5}x-3\right)\left(\frac{1}{5}x+3\right)$

Q5 Find the expansion of each of the following

1	$(2-3x)^2$	2	$(x-3)^2$	3	$(a+5)^2$
4	$(2ab+5)^2$	5	$(2x+3y)^2$	6	$(5x+4)^2$

Q5 Multiply each of the following

1	$(x+1)(x+2)(x+3)$	2	$(2x+3)(x^2+2x+3)$	3	$(x+2)(x^2-3x+1)$

Q6 Simplify each of the following

1	$(2x+3)^2-12x$ Then find the numerical value when $x=2$	2	$(x+2)^2-(x+2)(x-2)$ Then find the numerical value when $x=3$
3	$(2x+5)(2x-5)+25$ Then find the numerical value when $x=-1$	4	$(2n-1)^2-(2n+1)(2n-1)$ Then find the numerical value when $n=-3$

Q6	Solve each of the following equations in Q

1	$(2x+5)(2x-5)=11$	2	$(x-3)(x^2+3x+9)=0$	3	$(x-2)(x+2)=5$
4	$(x-3)(x+3)=9$	5	$(x+2)(x^2-2x+4)=0$	6	$(3x-2)(3x+2)=5$

lesson 7 Dividing an algebraic term or an algebraic expression by an algebraic limit

Dividing an algebraic term by algebraic term

- When dividing an algebraic term by another algebraic term, we divide the coefficients, and subtract the exponents of variables that have the same base
- For example: $(21X^4) \div (-3X^2) = -7X^2$
- Notice that : Dividing by zero has no meaning and therefore all variables divided by it are not equal to zero

Example 1

Find the quotient of each of the following

(1) $\frac{15x^3}{5x^2}$	(2) $(-20x^2) \div (5x)$	(3) $\frac{10a^5b^7}{2a^3b^2}$
$5a^2b^5$	$-4x$	$3x$
(4) $\frac{-9x^5y^4z^2}{3x^4y^4z}$	(5) $(-10a^3b^4c) \div (-2a^3b)$	(6) $\frac{12x^3y^2z}{3x^2yz}$
$-3x$	$5b^3c$	$4xy$

Notice

- In both the divisor and the divisor if the same variable is found with the same exponent, it is omitted, since the outside of their division is equal to 1

Exercise

Complete the following

(1) $\frac{8x^3y^4}{-4x^3y^2} = ...$	(2) $\frac{-15a^6}{-5a^2} = ...$	(3) $\frac{10x^3}{2x^2} = ...$
(4) $\frac{15a^3b^4c}{5a^2c} = ...$	(5) $(-8x^3y^5) \div (8x^2y^3) = ...$	(6) $\frac{12x^3y^2z}{4x^2y^2z} = ...$

Dividing an algebraic expression by an algebraic term

- For example :

$$\frac{8y^6 - 4y^3}{4y^2} = \frac{8y^6}{4y^2} - \frac{4y^3}{4y^2} = 2y^4 - y$$

- Notice that : $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ ، $\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$
- Therefore, when dividing an algebraic expression by an algebraic term, we will use the same method.

Example 2

Find the quotient of each of the following

(1) $\frac{8x^3-4x^2+2x}{2x}$	(2) $\frac{6x+4}{2}$	(3) $\frac{15x^2+10x}{5x}$
$=\frac{8x^3}{2x}-\frac{4x^2}{2x}+\frac{2x}{2x}=4x^2-2x+1$	$=\frac{6x}{2}+\frac{4}{2}=3x+2$	$=\frac{15x^2}{5x}+\frac{10x}{5x}=3x+2$
(4) $\frac{6x^3-3x^2}{-3x^2}$	(5) $\frac{8y^2x+6yx^2}{2yx}$	(6) $\frac{24x^2-18x^3-42x^2}{6x^2}$
$=\frac{6x^3}{-3x^2}-\frac{3x^2}{-3x^2}=-2x+1$	$=\frac{8y^2x}{2yx}+\frac{6yx}{2yx}=4y+3$	$\frac{24x^4}{6x^2}-\frac{18x^3}{6x^2}-\frac{42x^2}{6x^2}=6x^2-3x-7$
(7) $\frac{5a^2b+10ac}{5a}$	(8) $\frac{3ab^2+9a^2b-6a^2b^2}{3ab}$	(9) $\frac{12x^4+6x^3-2x^2}{-2x^2}$
$=\frac{5a^2b}{5a}+\frac{10ac}{5a}=ab+2c$	$\frac{3ab^2}{3ab}+\frac{9a^2b}{3ab}-\frac{6a^2b^2}{3ab}=b+3a-2ab$	$\frac{12x^4}{-2x^2}+\frac{6x^3}{-2x^2}-\frac{2x^2}{-2x^2}=-6x^2-3x+1$

Exercise

Find the quotient of each of the following

(1) $\frac{48x^3-80x^2}{8x^2}$	(2) $\frac{9a^3+3a^2}{-3a^2}$	(3) $\frac{18m^3+32m^2}{2m^2}$
(4) $\frac{8a^4+6a^3-4a^2}{-2a^2}$	(5) $\frac{6y^5-2y^4+y^3}{2y^3}$	(6) $\frac{18x^4y^5-42x^5y^4}{-6x^2y^2}$
(7) $\frac{8m^3n-4m^2}{4m^2}$	(8) $\frac{6a^3b^2-9b^2a^3+3a^2b^2}{3a^2b^2}$	(9) $\frac{a^3b^2+a^2b^3+ab}{ab}$

Example 3

Find the quotient of each of the following

(1) $\frac{9x^2(2x^2-4x+6)}{6x}$

$=\frac{18x^4-36x^3+54x^2}{6x}=\frac{18x^4}{6x}-\frac{36x^3}{6x}+\frac{54x^2}{6x}$

$=3x^3-6x^2+9x$

(2) $\frac{4x^3(2x^2-3x+1)}{2x}$

$=\frac{8x^5-12x^4+4x^3}{2x}=\frac{8x^5}{2x}-\frac{12x^4}{2x}+\frac{4x^3}{2x}$

$=4x^4-6x^3+2x^2$

Exercise

Find the quotient of each of the following

(1) $\frac{2x^3(4x^2+6x-12)}{4x^2}$

(2) $\frac{6a^2(2a^2+3a-1)}{2a}$

Example 4

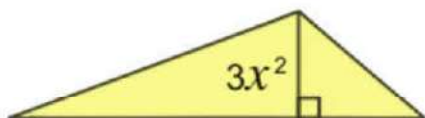
Answer the following questions

(1) If the area of a rectangle is $(6x^3+4x^2+2x)$ square unit , and one of its dimension is $2x$ length unit, find the other dimension in terms of x

$\because$ the area of the rectangle $=\ell\times w$

The other dimension $=\frac{6x^3+4x^2+2x}{2x}=\frac{6x^3}{2x}+\frac{4x^2}{2x}+\frac{2x}{2x}$

$=3x^2+2x+1$

(2) If the area of the triangle is $(6x^4+9x^3+3x^2)$ Square Unit ،And its height corresponding to this base is equal to $3x^2$ length unit, find the length of its base in terms of x Then calculate the numerical value of the base length at $x=5$

$\because$ The area of the triangle $=\frac{1}{2}$ base $\times$ height

$\therefore$ Base length $=\frac{\text{twice the area of the triangle}}{\text{height}}$

$\therefore$ Base length in terms of x

$=\frac{2(6x^4+9x^3+3x^2)}{3x^2}=\frac{12x^4+18x^3+6x^2}{3x^2}$

$=4x^2+6x+2$

The numerical value of the base length : $4(5)^2+6(5)+2=132$

Example 5

Simplify each of the following

(1) $\frac{a^3}{-a^2}+\frac{-4a^2}{a^2}-\frac{5a^4}{a^3}$

$-a-4-5a=-6a-4$

(2) $\frac{9x^3}{3x^2}+\frac{2x}{x}-\frac{3x^4}{x^3}$

$3\cancel{x}+2-3\cancel{x}=2$

(3) $\frac{48x^4-144x^3-96x^2}{-6x\times 8x}$

$=\frac{48x^4-144x^3-96x^2}{-48x^2}=\frac{48x^4}{-48x^2}-\frac{144x^3}{-48x^2}-\frac{96x^2}{-48x^2}$

$=-x^2+3x+2$

(4) $\frac{28x^2-42x}{7x}+\frac{14x^2-35x}{-7x}$

$=\frac{28x^2}{7x}-\frac{42x}{7x}+\frac{14x^2}{-7x}-\frac{35x}{-7x}=$

$=4x-6-2x+5=2x-1$

Home work — Dividing an algebraic term or an algebraic expression by an algebraic limit

Q1 Choose the correct answer

1. If : $\frac{8x^2}{a}=1$ then $a=$ ....
   (a) -1 (b) 1 (c) $8x^2$ (d) $-8x^2$

2. $(x^3+x^2)\div x^2=$ ....
   (a) 0 (b) x (c) $x+1$ (d) $2x+1$

3. $\frac{3x^2-6x}{3x}=$ ....
   (a) $-x$ (b) $-x^2$ (c) x^2-2x (d) $x-2$

4. $12y\div(-3y)=$ ......
   (a) $4y^2$ (b) -4 (c) $9y^2$ (d) $-4y$

5. $10a^5b^4\div$ .... $=2a^2b^3$
   (a) $2a^3b^2$ (b) $-2a^2b$ (c) ab^4 (d) $-3ab$

6. $\frac{y^5}{y^2}+y^3=$ ....
   (a) y^6 (b) y^3 (c) $2y^3$ (d) y^{10}

7. $24x^3\div(-6x^2)=$ ....
   (a) $-4x$ (b) -4 (c) $-4x^5$ (d) $-4x^2$

8. $\frac{a+b}{c}=$ ....
   (a) $a+\frac{b}{c}$ (b) $\frac{a}{c}+b$ (c) $\frac{a}{c}+\frac{b}{c}$ (d) $\frac{ab}{c}$

9. .... $\div(-2x^2y)=12xy^2$
   (a) $6xy$ (b) $-6xy$ (c) $24x^3y^3$ (d) $-24x^3y^3$

10. $(35x^2y+5y^2)\div 5y=$ ....
    (a) $7x+5y$ (b) $7xy+y$ (c) $7x^2+5y$ (d) $7x^2+y$

11. $\frac{14a^3-7a}{-7a}=$ ....
    (a) $2a-1$ (b) $-2a^2+1$ (c) $2a+1$ (d) $2a^2-1$

Q2 Find the quotient of each of the following

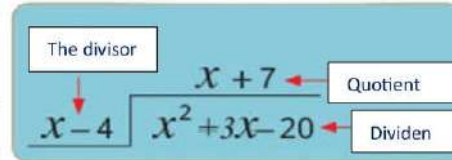
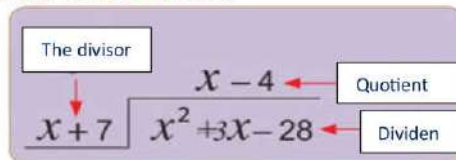
1	$(-18x^6)\div(-6x^5)$	2	$(-8a^4)\div(-2a)$	3	$(18x^5)\div(9x^3)$
4	$(21a^3b^2c)\div(7a^2b^2c)$	5	$(-15y^3x^2)\div(5y^2)$	6	$(10a^3b^2)\div(5a^2b)$
7	$(2x-4x^2+8x^3)\div(2x)$	8	$(y+y^3-y^2)\div(-y)$	9	$(x^3-x^2+x)\div(x)$

Q3	Simplify each of the following

1	$\frac{8x^2+6x}{2x}+\frac{6x^2-9x}{3x}$	2	$\frac{4a^2}{2a}+\frac{2a}{2}+\frac{-6a^3}{3a^2}$	3	$\frac{x^2}{-x}+\frac{-4x}{x}-\frac{3x^3}{x^2}$
4	$\frac{36x^5-72x^4+48x^2}{-2x\times 6x}$			5	$\frac{2x\left(6x^2-2x+8\right)}{4x}$

Lesson 8 Dividing Algebraic Expressions

Notice that

- When multiplying algebraic expressions: $(x+7)(x-4)=x^2+3x-28$
- Since division is an inverse process of multiplication, the following can be deduced: :
  - $(x^2+3x-28)\div(x+7)=x-4$
  - $(x^2+3x-28)\div(x-4)=x+7$
- The division process can be written as follows:

Example 1

Answer the following questions

Find the quotient of (x^2+5x+6) by $(x+3)$

- Divide x^2 by x then the result is x
- Multiply x by $(x+3)$ then the result is
- Subtract x^2+3x from x^2+5x+6 you get
- So the quotient is $x+2$

(2) Find the quotient of (x^2+x-12) by $(x+4)$

$$\begin{array}{r|l} & x-3 \\ x+4 & x^2+x-12 \\ & \underline{x^2+4x} \\ & -3x-12 \\ & \underline{-3x-12} \\ & 0 \quad 0 \end{array}$$

the quotient is $x-3$

(3) Find the quotient of $(x^2-2x-10)$ by $(x-5)$

$$\begin{array}{r|l} & x+3 \\ x-5 & x^2-2x-10 \\ & \underline{x^2-5x} \\ & 3x-10 \\ & \underline{3x-10} \\ & 0 \quad 0 \end{array}$$

the quotient is $x+3$

Important remarks

- Similar terms must be written under each other during the division process
- Before starting the division process, the algebraic terms must be arranged in both the divisor and the divisor ascending or descending according to the powers (exponents) of the variable

Example 2

Answer the following questions

Find the quotient of $(2x + x^3 - 12)$ by $(x - 2)$

$$\begin{array}{r|l} & x^2 + 2x + 6 \\ x - 2 & x^3 \quad\quad + 2x - 12 \\ & x^3 - 2x^2 \\ & 2x^2 + 2x - 12 \\ & 2x^2 - 4x \\ & 6x - 12 \\ & 6x - 12 \\ & 0 \quad 0 \end{array}$$

- Write the divisor after arranging its terms descending according to the powers of x as follows $x^3 + 2x - 12$
- And because there is no limit that includes X^2 we leave a blank space for it
- the quotient is $x^2 + 2x + 6$

Example 3

Answer the following questions

(1) A rectangle with an area of $(x^2 + 8x + 15)$ length unit and one of its dimension is $(x + 3)$ length unit. find its other dimension

$$\begin{array}{r|l} & x + 5 \\ x + 3 & x^2 + 8x + 15 \\ & x^2 + 3x \\ & 5x + 15 \\ & 5x + 15 \\ & 0 \quad 0 \end{array}$$

(2) If $(2x - 3)$ One of the factors of the expression $(2x^2 + 7x - 15)$ Find the other factor

$$\begin{array}{r|l} & x + 5 \\ 2x - 3 & 2x^2 + 7x - 15 \\ & 2x^2 - 3x \\ & 10x - 15 \\ & 10x - 15 \\ & 0 \quad 0 \end{array}$$

the other factor $x + 5$

Exercise

Answer the following questions

(1) Find the quotient of $(x^2 - 9x + 14)$ by $(x - 7)$

(2) Find the quotient of $(x^2 + 9x + 20)$ by $(x + 5)$

(3) Find the quotient of $(x^2 - x - 20)$ by $(x - 5)$

(4) Find the quotient of $(x^2 + 4x - 12)$ by $(x + 6)$

(5) Find the quotient of $(x^3 - x^2 - 2x + 8)$ by $(x^2 - 3x + 4)$

(6) Find the quotient of $(2x^2 + 7x + 3)$ by $(x + 3)$

Example 4

Answer the following question

(1) If the expression $(2x^2+9x+k)$ Divisible by $2x+1$ find the value of k

$$\begin{array}{r|l} & x+4 \\ 2x+1 & 2x^2+9x+K \\ & \ominus 2x^2 \ominus X \\ & 8x+K \\ & \ominus 8x \ominus 4 \\ & K-4 \end{array}$$

$\therefore K-4=0 \qquad \therefore K=4$

Exercise

Answer the following question

(2) If the expression $(3x^2-10x+k)$ Divisible by $3x-1$ find the value of k

Dividing Algebraic Expressions — Home work

Q1 Choose the correct answer from the given answers

1 If $\frac{3x+15}{x-a}=3$ what is the value of a?
(a) -5 (b) -3 (c) 3 (d) 5

2 If $\frac{2x+a}{x+3}=2$ what is the value of a ?
(a) 2 (b) 3 (c) 5 (d) 6

3 If it is out of division (x^3-4x) by $(x-2)$ is $(ax+x^2)$ what is the value of a ?
(a) -4 (b) -2 (c) 2 (d) 4

4 If $\frac{x-2}{2-x}=a$ what is the value of a ?
(a) -2 (b) -1 (c) 1 (d) 2

5 If $(x-2)$ is one of the factors of the expression (x^2-x+2), then the other factor is ?
(a) x (b) $x+2$
(c) $x-1$ (d) $x+1$

6 If it is out of division $(x^2-2x-35)$ by $(x+5)$ is $(x+b)$ what is the value of b ?
(a) -7 (b) -5 (c) 5 (d) 7

7 If $\frac{a}{x-5}=-1$ then a = ...
(a) -1 (b) $x+5$
(c) $5-x$ (d) $-x-5$

8 a rectangle with area (x^2+6x+8) square unit and its length $(x+4)$ length unit $(x+b)$ its width = ...
(a) x (b) $x+2$
(c) $x-2$ (d) $x-4$

Q2 Find the quotient of

1	$x^2 - x - 12$ by $x + 4$	2	$x^2 - 8x + 12$ by $x - 6$	3	$x^2 + 9x + 20$ by $x + 4$
4	$3x^2 - x - 2$ by $3x + 2$	5	$x + x^2 - 2$ by $x - 1$	6	$7x + 10 + x^2$ by $x + 2$
7	$x^3 - 27$ by $x - 3$	8	$7x - 5x^2 + 2x^3 - 6$ by $2x - 3$	9	$3x^2 + 1 - 4x$ by $x - 1$
10	$x^2 - 9$ by $x - 3$	11	$x^4 - 1$ by $x^2 + 1$	12	$x^3 + 8$ by $x^2 - 2x + 4$

Q3 Answer the following questions

1	Divide $(-3x^2 + x^3 - x + 6)$ by $(x - 2)$ then find the numerical value of the result when $x = 3$	2	Divide $x^2 - 5x - 6$ by $x + 1$ then find the numerical value of the result when $x = 7$
3	If $x - 2$ one of the factors of the algebraic expression $x^3 - 8$, find the other factor	4	If $x - 6$ one of the factors of the algebraic expression $x^2 - 9x + 18$, find the other factor
5	If the algebraic expression $2x^2 + 7x + a$ is divisible by $x + 3$, find the value of a	6	a rectangle with area $(x^2 + 9x + 20)$ square units, its length is $(X + 5)$ length unit . find its perimeter when $x = 2$

LESSON 9 Areas

Notice that

⊙ Symbols commonly used in calculating the perimeter and area of some geometric shapes :

Area	Perimeter	Side length of a square or rhombus	Rectangle length	Rectangle width	Height	Base length
A	P	s	ℓ	w	h	b

Remember

Mathematical formulas for the area and perimeter of some geometric shapes

Parallelogram

Perimeter: $P = 2(b_1 + b_2)$

Area: $A = b_1 \times h$

Rectangle

Perimeter: $P = 2(\ell + w)$

Area: $A = \ell \times w$

square

Perimeter: $P = 4s$

Area: $A = s^2$

triangle

Perimeter: $P = a + b + c$

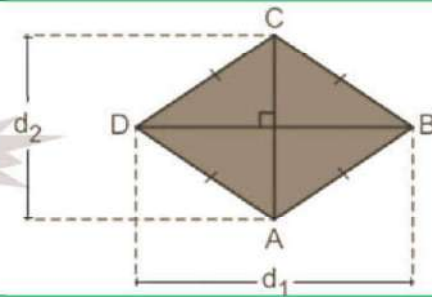
Area: $A = \frac{1}{2} b \times h$

rhombus

Perimeter: $P = 4s$

area : $A = s \times h$

Area of rhombus given diagonal

- The area of the rhombus $= \frac{1}{2}$ The product of the lengths of its diagonals
- Let area A and the lengths of the two diagonals d_2 ، d_1 then : $A = \frac{1}{2} \times d_1 \times d_2$

Remember

- A rhombus is a parallelogram with two adjacent sides of equal length.
- The sides of the rhombus are equal in length
- The diagonals of the rhombus are perpendicular and bisect each other

Remember

Units of length measurement in the English system	Units of length measurement in the metric system
1 foot = 12 inches	1 cm = 10 mm
1 yard = 36 inches = 3 feet	1 decm = 10 cm
1 Mile = 5280 Foot	1 meter = 100 cm
	1km = 1000 meters

Example 1

Answer the following questions

(1)A rhombus whose diagonals are 10 cm and 12 cm find its area

The area of the rhombus $= \frac{1}{2}$ The product of the lengths of its diagonals

$A = \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 10 \times 12 = 60$

$\therefore$ the area of the rhombus is 60 cm^2

(2)A rhombus has an area of 24 cm^2 and the length of one diagonal is 8 cm Find the length of the other diagonal

$\because A = \frac{1}{2} \times d_1 \times d_2$ $\therefore 24 = \frac{1}{2} \times 8 \times d_2$

$\therefore 24 = 4d_2$ $\therefore d_2 = \frac{24}{4} = 6$

$\therefore$ The length of the other diagonal = 6 cm

example 1

(3)A rhombus whose diagonals are 8 cm and 14 cm and its height is 10 cm , find its area

$\because A = \frac{1}{2} \times d_1 \times d_2 \Rightarrow \therefore A = \frac{1}{2} \times 8 \times 14 = 56$

$\therefore \text{side length} = \frac{\text{the area}}{\text{the height}} = \frac{56}{10} = 5.6\text{cm}$

$\because$ The rhombus perimeter $= 4 \times \ell$

$= 4 \times 5.6 = 22.4\text{cm}$

(3)A rhombus with a side length of 12 feet, a height of 8 feet, and a length of one diagonal of 20 feet. Find the length of the other diagonal

The area of the rhombus $\ell \times h = 8 \times 12 = 96 \text{ ft}^2$

$\because A = \frac{1}{2} \times d_1 \times d_2$

$\therefore 96 = \frac{1}{2} \times 20 \times d_2 \Rightarrow 96 = 10d_2$

$\therefore d_2 = \frac{96}{10} = 9.6$

$\therefore$ the length of the other diagonal = 9.6 feet

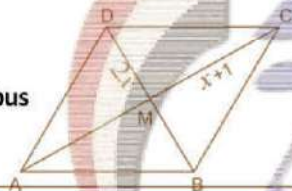
Example 2

Answer the following question

ABCD is a rhombus its two diagonals intersect at M,

$MD = 2x$, $MC = x + 1$

Find the area of the rhombus

in terms of X

$\because$ the two diagonals in the rhombus bisects each other

$\therefore BD = 2DM = 2(2x) = 4x$

$\therefore AC = 2CM = 2(x+1)$

$\therefore A = \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times (4x) \times 2(x+1)$

$= (4x)(x+1) = 4x^2 + 4x$

Exercise

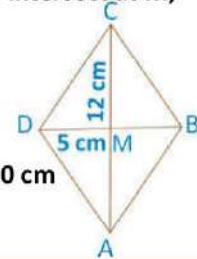
Answer the following question

ABCD is a rhombus its two diagonals intersect at M,

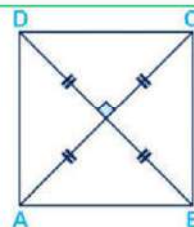
$MD = 5$, $MC = 12$

Find the area of the rhombus

and if the height of the rhombus is 10 cm

Find its perimeter

Area of the square given the diagonal length

⊙ A square is a rhombus with two diameters of equal length, thus

, The area of the square $= \frac{1}{2}$ the diagonal length $\times$ the diagonal length $= \frac{1}{2}d^2$

Example 3

Answer the following questions

(1) A square with a diagonal length of 10 cm find its area

$\because$ The area of the square $= \frac{1}{2}d^2$

$\therefore A = \frac{1}{2}(10)^2 = \frac{1}{2} \times 100 = 50$

(2)If the area of a square is 18 cm², find the length of its diagonal

$\because$ The area of the square $= \frac{1}{2}d^2$

$\therefore 18 = \frac{1}{2}(d)^2 \Rightarrow d^2 = 36$

$\therefore d = \sqrt{36} = 6$

(3)Find the length of the diameter of the square whose area is equal to the area of a rhombus with a diagonal length of 9 cm, 4 cm

$\because$ The area of the rhombus $= \frac{1}{2} \times d_1 \times d_2$

$\therefore$ The area of the rhombus $= 4 \times 9 \times \frac{1}{2} = 18 \text{ cm}^2$

$\because$ The area of the square $= \frac{1}{2}d^2$

the area of the rhombus equal the area of the square

$\therefore 18 = \frac{1}{2}(d)^2 \Rightarrow d^2 = 36$

$\therefore d = \sqrt{36} = 6$

(4)which is greater in area, a square its diagonal length is 12 cm or a rectangle of length of 12 cm and a width of 10 cm .

Exercise

Answer the following questions

(1)A square its diagonal length is $(2x)$ length unit Find its area in terms of X

(2)If the area of a square is 50 cm², find the length of its diagonal

(3)Find the length of the diameter of the square whose area is equal to the area of a rhombus with a diagonal length of 10 cm, 6.4 cm

(4)which is greater in area, a square its diagonal length is 14 cm or a rhombus its two diagonal length are 12 cm, 18 cm

example 3

$\because$ the area of the square $= \frac{1}{2} d^2$

$= \frac{1}{2} \times (12)^2 = 72\, cm^2$

, the area of the rectangle $= \ell \times w$

$= 10 \times 12 = 120\, cm^2$

$\therefore$ the area of the rectangle $>$ the area of the square

Exercise

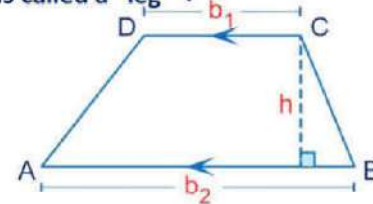
The trapezium area

⊙ A trapezium is a quadrilateral with only two parallel sides

⊙ Each side of the parallel side is called a "base" and each of the two non-parallel sides is called a "leg ".

In the opposite figure :

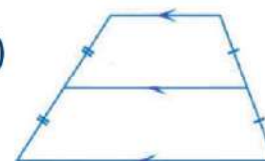
⊙ ABCD is a trapezium $\overline{AB}$ is its great base $\overline{DC}$ ، its small base

⊙ each of $\overline{BC}$, $\overline{AD}$ is a leg ⊙ h is the height of the trapezium

⊙ The area of the trapezium $= \frac{1}{2}$ The sum of lengths of the two parallel $\times$ height

⊙ The area of the trapezium $= \frac{1}{2}(b_1 + b_2) \times h$

Notice that

⊙ The middle base of the trapezium is the line segment joining the two midpoints of the two legs of the trapezium

⊙ The length of the middle base $\frac{1}{2} =$ the sum of lengths of the two parallel bases $= \frac{1}{2}(b_1 + b_2)$

Example 4

(1) A trapezium in which the lengths of the two parallel bases are 8 cm and 10 cm . and its height is 6 cm .Find its area

$\because A = \frac{1}{2}(b_1 + b_2) \times h$

$\therefore A = \frac{1}{2}(8 + 10) \times 6 = 54$

(2)A trapezium in which the length of its middle base equals 10 cm and its height is 14 cm . find its area

$\because$ The area of the trapezium $=$ the length of the middle base $\times$ height

$= 10 \times 12 = 120\, cm^2$

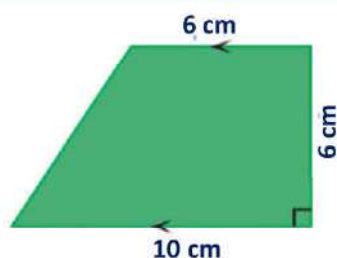
Exercise

(1)A trapezium in which the length of its middle base equals 12.5 cm and its height is 8 cm . find its area

(2 A trapezium in which the lengths of the two parallel bases are 12 cm and 14 cm . and its height is 10 cm .Find its area

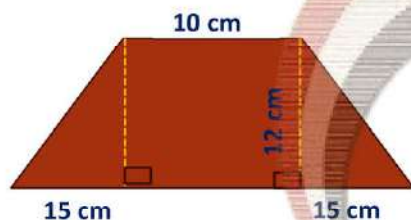
Example 4

Find the trapezium area in each of the following figures

(1)

$\because A = \frac{1}{2}(b_1 + b_2) \times h$

$\therefore A = \frac{1}{2}(6 + 10) \times 6 = 48$

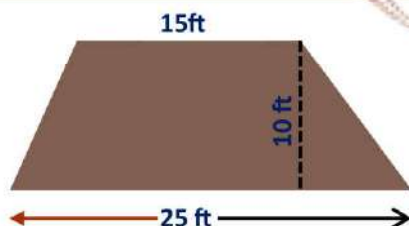
The area of the trapezium $= 48 cm^2$

(2)

The length of the great base $40 = 15 + 10 + 15 =$

$\because A = \frac{1}{2}(b_1 + b_2) \times h$

$\therefore A = \frac{1}{2}(10 + 40) \times 12 = 300$

The area of the trapezium $= 300 cm^2$

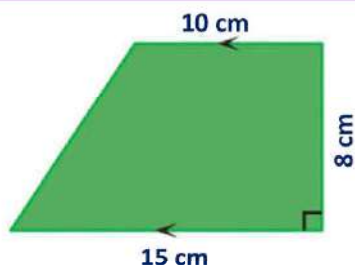
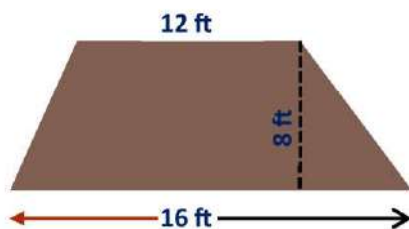
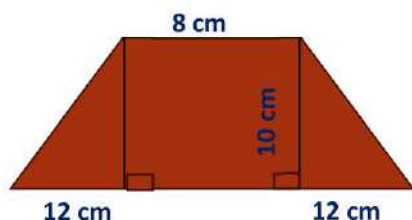
(3)

$\because A = \frac{1}{2}(b_1 + b_2) \times h$

$\therefore A = \frac{1}{2}(15 + 25) \times 10 = 200$

Exercise

(1)

Exercise

Example 5

Answer the following questions

(1)A trapezium in which its area is 56 cm^2 and if the length of the bases 6 cm, 8 cm find its height

$\because A = \frac{1}{2}(b_1 + b_2) \times h$

$\therefore 56 = \frac{1}{2}(6 + 8) \times h$

$\therefore 56 = 7 \times h \quad \Rightarrow \quad \therefore h = 8$

$\therefore$ The height of the trapezium $= 8$cm

(2)trapezoid area of 120 cm^2 If the length of its smaller base 6 cm and height 10 cm find the length of its great base

$\because A = \frac{1}{2}(b_1 + b_2) \times h$

$\therefore 120 = \frac{1}{2}(6 + b_2) \times 10$

$\therefore 120 = 5(6 + b_2) \quad \Rightarrow \quad \therefore 6 + b_2 = 24$

$\therefore b_2 = 24 - 6 = 18$

$\therefore$ The length of the great base $= 18$cm

Home work | **Areas**

Q1 Choose the correct answer from the given answers

1 If the area of a square is 450 square units, what is the length of its diagonal in units of length ?
(a)15 (b)30 (c)45 (d)90

2 If the area of a rhombus is 100 square units, what is the product of the diagonal longitudinal ?
(a)25 (b)50 (c)100 (d)200

3 A rhombus with a diagonal length of 8 cm, 6 cm with an area of ...... cm^2
(a)8 (b)6 (c)48 (d)24

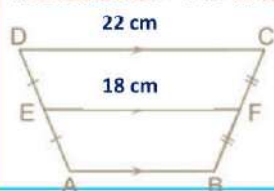
4 Trapezium the sum of its two bases lengths is 16 cm and its height is 5 cm, what is its area in centimeters?
(a)20 (b)40 (c)80 (d)160

5 A trapezoid with an area of 15 cm2 and a height of 3 cm, what is the length of its middle base ?
(a)45 (b)18 (c)10 (d)5

6 A square with a diagonal of 8 cm with an area of .... cm^2
(a)8 (b)16 32 (c) (d)64

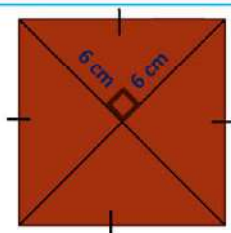
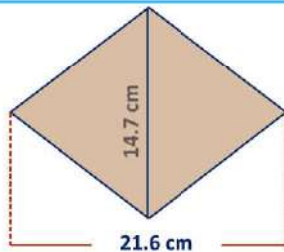
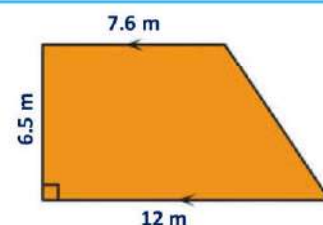
7 A square with side length s and area A How much is the area of a square with a diagonal length of 2s ?
(a)A (b)2A (c)4A (d)A^2

8 A trapezium whose two parallel bases are 6 cm long, 8 cm has a medium base length .... cm
(a)48 (b)24 (c)7 (d)14

9 The length of؟ $\overline{AB}$ in centimeters ?

(a)14 (b)20 (c)26 (d)28

10 A square with an area of 72 cm^2, what is the length of its diagonal ?
(a)36 (b)72 (c)12 (d)14

Q2 Find the area of each of the following figures

1

2

3

Q3 Answer the following questions

1 A trapezium whose parallel bases are 14 cm, 18 cm and its height is 12 cm high. Find its area

2 A trapezium with an area of 175 square meters and a length of two parallel bases of 15 meters and 21 meters. Find its height

3 A trapezium has an area of 225 square inches, one of its parallel bases is 23 inches long, and its height is 7.5 inches.find the length of the other base

4 A trapezium has an area of 36 cm^2, if its base is 8 cm long and its height is 6 cm .Find the length of its smaller base .

5 A square of area 72 cm^2 Find the length of its diagonal

6 Find the area of a rhombus with a circumference of 52 cm, and the length of one of its diagonals is 10 cm

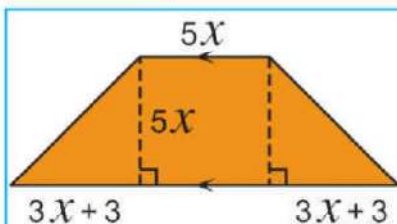
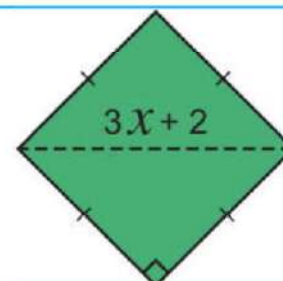
7 Trapezium area of 180 cm^2 and height 12 cm and the ratio between the length of the bases 2: 3 what is the length of each of them

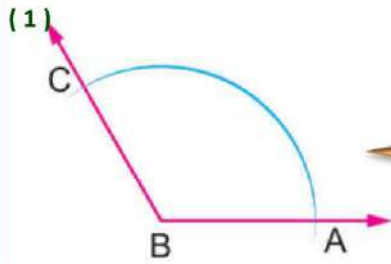
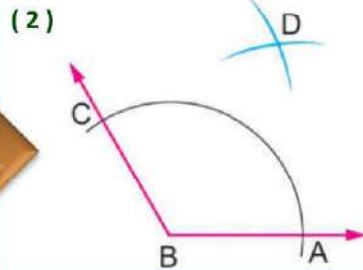
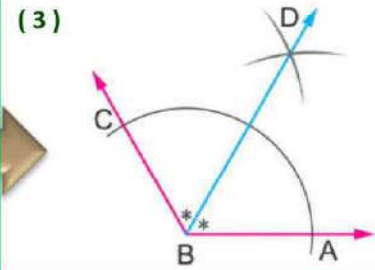
8 Find the area of a rhombus the lengths of its diagonals are 14 cm, 18 cm

Q4 In terms of x, find the area of each of the following two figures, and then find the numerical value of the area at x = 4

1

2

Lesson 10 Geometrical Constructions

Constructing the bisector of a given angle

⊙ To bisect a given angle we follow the next steps

With B as a centre and a suitable radius , draw an arc that intersects The two sides of the angle B at A and C

Taking A and C as centers and using the compass with a suitable radius , draw two arcs to intersect at the point D

Draw $\overrightarrow{BD}$ to be the bisector of $\angle B$ then $m(\angle ABD) = m(\angle CBD)$

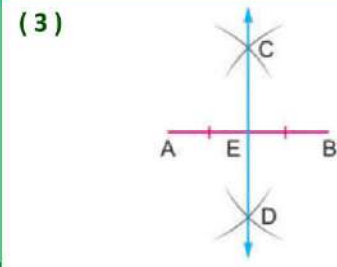
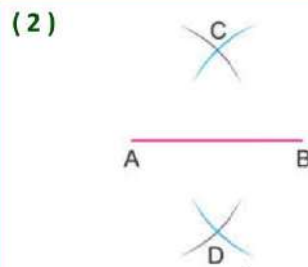
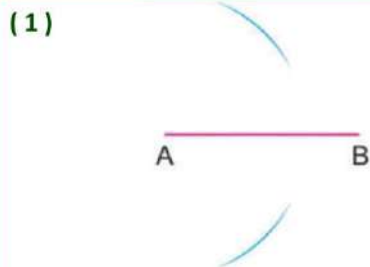
Exercise

Answer the following questions

(1) Draw an angle with measure 120° and bisect it using ruler and compass

(2) Draw an angle with measure 80° and bisect it using ruler and compass

Bisecting a given line segment

⊙ Bisecting a given line segment

Using a compass at A as a centre and with a radius greater than $\frac{1}{2}\overline{AB}$, draw two arcs in the opposite sides of $\overline{AB}$

Using the compass at B as a centre and with the same radius , draw two other arcs to intersect the previous two arcs at the two points C and D

Draw $\overleftrightarrow{CD}$ to cut $\overline{AB}$ at the point E which is the midpoint of $\overline{AB}$

Exercise

Answer the following questions

(1) using the ruler and the compass , draw the line segment $\overline{CD}$ with the length 9 cm and bisect it

(2) using the ruler and the compass , draw the line segment $\overline{AB}$ with the length 7 cm and bisect it

Triangle drawing

First — Draw a triangle given the lengths of its sides

Draw the triangle in which $AB = 5cm$, $AC = 3cm$ and $Bc = 4cm$

(1)	(2)	(3)
A — 5 cm — B	A — 5 cm — B	C; A — 5 cm — B
Use a ruler and draw a line segment $\overline{AB}$ 5 cm long	Open the compass with a radius 4 cm, focus on the point B and draw an arc	open the compass with a radius 3 cm and focus on the point A and draw an arc intersect the first arc at the point C then draw $\overline{AC}$, $\overline{BC}$

Exercise

Draw the triangle in which
$AB = 5cm$, $AC = 6cm$ and $Bc = 7cm$

Second — Draw a triangle given the lengths of two sides and measure the angle between them

⊙ To draw the triangle in which the length of $AB = 5\,cm$ $m(\angle BAC) = 70^\circ$ $AC = 4\,cm$ طول we follow the next steps

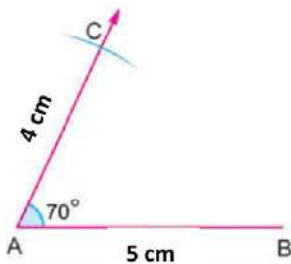
(1)

A 5 cm B

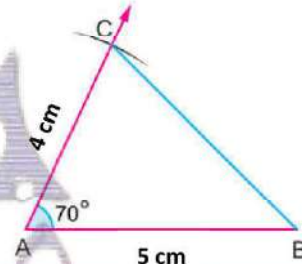
Use the ruler and draw a line segment $\overline{AB}$ with length 5 cm

(2)

Use the protractor, and from point A Measure an angle of 70° Then draw a ray that defines the angle

(4)

Open the caliper with a radius of 4 cm, then focus on A
So it And draw an arc that cuts the ray drawn at a point C
equal 4 cm will be the length of $\overline{AC}$

(3)

C 4 cm 70° A 5 cm B

Draw BC to get the triangle

Exercise

Draw the triangle in which
$m(A) = 80^\circ$, $5 = AC = AB$

Third — Drawing a triangle given the measure of two angles and the length of the side drawn between their vertices

⊙ To draw the triangle in which $\overline{AB}$ equal 5 cm ، $m(\angle B) = 50^\circ$, $m(\angle A) = 60^\circ$ we follow the next steps

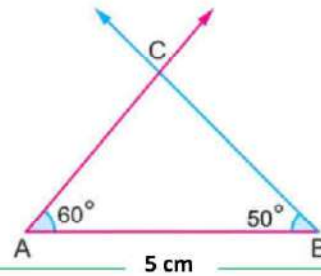
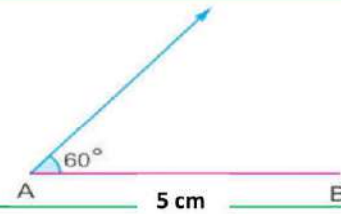
(1) Use the ruler and draw a line segment $\overline{AB}$ with length 5 cm

(2) From the point A determine an angle with measure 60° them draw the ray determine the angle

A 5 cm B

60°
A 5 cm B

(3) From the point B determine angle with measure 50° Then draw a ray that determines the angle and cuts the first ray in C then we get the triangle ABC

C
60° 50°
A 5 cm B

Exercise

Draw the triangle in which the length of $\overline{AB}$ equal 4 cm , $m(\angle B)=70^\circ$, $m(\angle C)=55^\circ$ Using measurement, determine the type of triangle relative to its sides

Home work | Geometrical Constructions

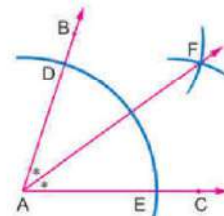
Q1 Choose the correct answer

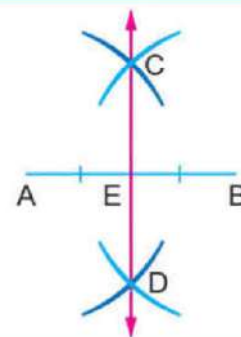
1 when bisecting $\angle BAC$ by compass , we find that :

(1) $m(\angle BAF) = ...$

(a) $m(\angle BFA)$ (b) $m(\angle EAF)$ (c) $m(\angle EFA)$ (d) $m(\angle BAC)$

(2) The length of $\overline{EF}$ must be equal to the length of ....

(a) $\overline{DF}$ (b) $\overline{AD}$ (c) $\overline{AE}$ (d) $\overline{AF}$

2 When bisecting a line segment $\overline{AB}$ with compass ,It should be ......

(a) $AC < \frac{1}{2}AB$ (b) $AC < AD$ (c) $AC > \frac{1}{2}AB$ (d) $AC < AE$

Q3 Answer the following questions

1 Draw $\overline{AB}$ with length 7 cm and then bisect using a ruler and a compass at the point C With an explanation of the steps of the solution, make sure with the ruler that C the midpoint of $\overline{AB}$

2 Draw $\angle ABC$ with measure 120° then bisect it using ruler and compass with explanation of the steps of the solution

3 Draw ΔABC in which :

$m(\angle C) = 55^\circ$ ، $m(\angle B) = 70^\circ$ ، $BC = 5cm$ Then determine by measure the type of triangle relative to the lengths of its sides

4 Draw ΔABC in which :

$AB = 7cm$ ، $BC = 5cm$ ، $m(\angle ABC) = 80^\circ$ Then determine the type of triangle relative to the measures of its angles

5 Draw an equilateral triangle ΔABC with a side length of 6 cm

6 Draw ΔXYZ in which :

$5 = XZ$, $4 = YZ$, $6 = XY$ Then determine by measure the type of triangle in relative to the measures of its angles

7

Draw ΔABC In which :

$m(\angle B) = 50^\circ$, $m(\angle A) = 70^\circ$, $8 = AB$ then bisects $\overline{AC}$ at the point D and bisects $\overline{BC}$ at the point D

then prove by measure : $AB = 2DE$

8

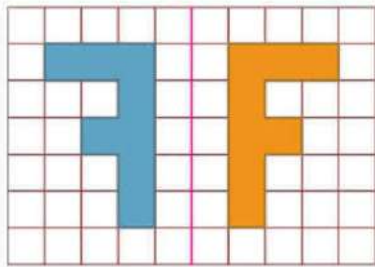
Draw $\angle ABC$ with measure 60° then bisect it using ruler and compass by the bisector $\overrightarrow{BD}$, then bisect $\angle CBD$, $\angle ABD$ with the bisectors $\overrightarrow{BF}$, $\overrightarrow{BE}$ respectively

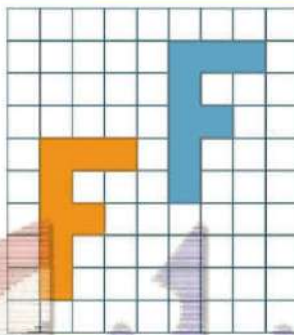
then prove by measuring that : $m(\angle ABF) = 3m(\angle CBF)$

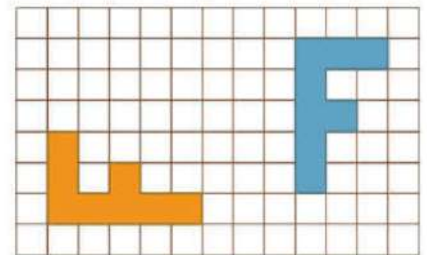
Lesson 11 Geometrical transformation

Geometrical transformation

If all the points of the geometric shape move according to a specific system, we get an image of this shape in a new position, it is said that this shape is under the influence of a geometric transformation, and examples of geometric transformations are reflection, transition and rotation

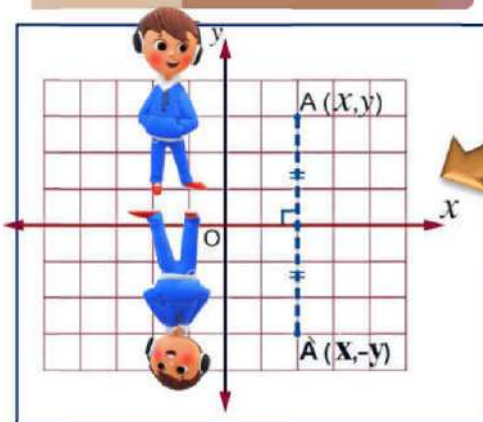
Reflection in a straight line

- Reflection in a straight is the formation of a mirror image of a shape across a line called the axis of reflection

transition

- Transition is the displacement of a shape on a straight line a specified distance and in a specified direction

Rotation

Rotation is a geometric transformation that turns the geometric figure around a fixed point with a certain angle in a certain direction

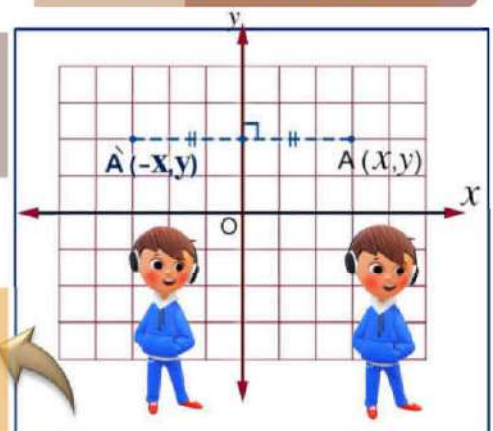
First Reflection in the two coordinate axes

The point and its reflection in a line are on two equal dimensions of that line, which is called the axis of reflection

Reflection in the x-axis

the image of the point $(1, -3)$ by reflection in the x – axis is $(1, 3)$

$$A(x, y) \xrightarrow[\text{x - axis}]{\text{by reflection on}} A'(x, -y)$$

Reflection in the y-axis

the image of the point $(-2, 3)$ by reflection in the y – axis is $(2, 3)$

$$A(x, y) \xrightarrow[\text{y - axis}]{\text{by reflection on}} A'(-x, y)$$

Notice that

(1)If the point lies on the x-axis, then its image reflected on the x-axis is the same.

For example : the image of the point $(1, 0)$ by reflection in the x- axis is $(1, 0)$

(2)If the point lies on the y-axis, then its image reflected on the y-axis is the same.

For example : the image of the point $(0, -3)$ by reflection in the y- axis is $(0, 3)$

Example 1

Complete the following

(1) The image of the point $(-1, -5)$ by reflection in the x – axis is ....

(2) The image of the point $(1, 2)$ by reflection in the x – axis is ....

(3) The image of the point $(-2, -1)$ by reflection in the y – axis is ....

(4) The image of the point $(-2, 5)$ by reflection on the y – axis is

(5) The image of the point $(-5, 0)$ by reflection in the x – axis is ....

(6) The image of the point $(0, 5)$ by reflection in the x – axis is ....

(7) The image of the point $(6, 0)$ by reflection in the y – axis is ....

(8) The image of the point $(-2, -1)$ by reflection in the y – axis is ....

(9) The image of the point $(-4, 0)$ is it self by reflection on ....

(10) the point $(-2, -1)$ is the image of the point $(2, -1)$ by reflection on the ....axis

(11) The image of the point $(0, 5)$ is it self by reflection on ....

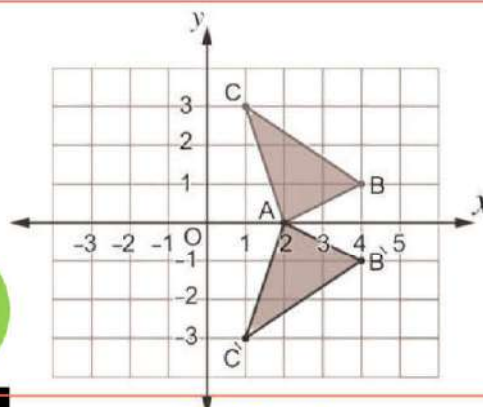
(12) The point .... is the image of the point $(-2, -1)$ by reflection on the y - axis

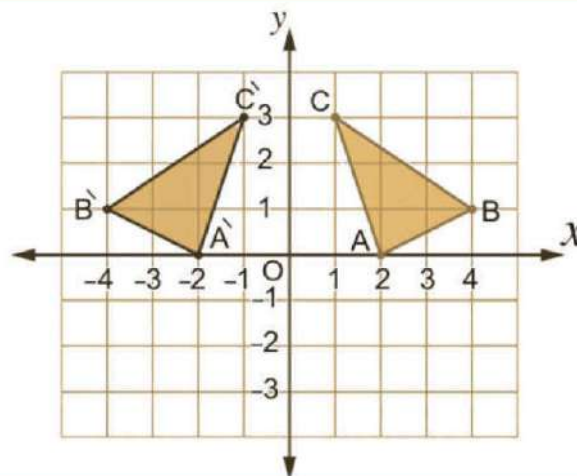
(13) the point $(-2, 1)$ is the image of the point $(2, -1)$ by reflection on the ....axis

(14) The point .... is the image of the point $(5, 3)$ by reflection on the x - axis

Example 2

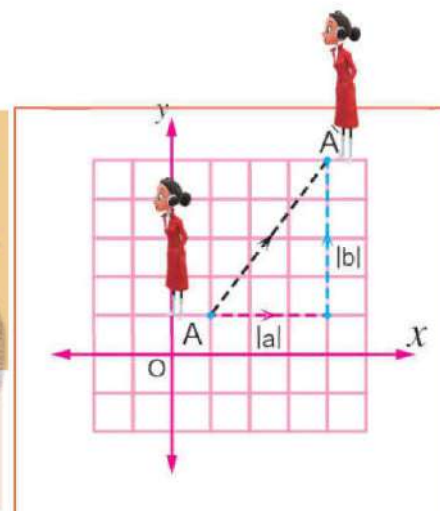
Draw the triangle ABC where $A(2, 0)$, $B(4, 1)$, $C(1, 3)$, then draw its image by reflection in (1) x- axis (2) y - axis

The point	Its image by reflection on x - axis
$A(2, 0)$	$A'(2, 0)$
$B(4, 1)$	$B'(4, -1)$
$C(1, 3)$	$C'(1, -3)$

Example 2

The point	Its image by reflection on y - axis
$A(2, 0)$	$A'(-2, 0)$
$B(4, 1)$	$B'(-4, 1)$
$C(1, 3)$	$C'(-1, 3)$

Second transition

Transition in the orthogonal Cartesian coordinates plane transforms each point by a displacement in the direction of the x - axis followed by a displacement in the direction of the y - axis

i . e The image of the point $A(x, y)$ by transition (a, b) is the point $A'(x+a, y+b)$

For example : The image of the point $A(2, 5)$ by transition 4 units to the right and 3 units upward i.e. by transition $(4, 3)$ is the point $A'(2+4, 5+3) = A'(6, 8)$

Notice that

(1) In the ordered pair $(4, 3)$ which express the transition

⊙ If a is positive, it means a transition to the right. If a is negative, it means a transition to the left.

⊙ If b is positive, it means an upward transition, and if b is negative, it means a downward transition.

For example : The image of the point $(6, 3)$ by transition two units to the left is $(6-2, 3) = (4, 3)$

The image of the point $(6, 3)$ by transition 4 units downward is $(6, 3-4) = (6, -1)$

(2) image = point + transition

For example : the image of the point $(2, 3)$ by transition $(1, 4)$ is $(2, 3)+(1, 4) = (3, 7)$

(2) point = image – transition

For example : the point which its image is $(5, 6)$ by transition $(3, 1)$ is $(5, 6)-(3, 1) = (2, 5)$

(3) transition = image – transition

for example : the point which translate $(1, 5)$ to the point $(4, 2)$ is $(4, 2)-(1, 5)=(3, -3)$

Example 3

Complete the following

(1) The image of the point $(3, 1)$ by transition 4 units to the left and 3 units upward is ....

(2) The image of the point $(1, 2)$ by transition 3 units to the right and two units upward is ....

(3) The image of the point $(-1, 2)$ by transition 5 units in negative direction of y – axis ....

(4) The image of the point $(-2, 5)$ by transition 5 units in the positive direction of x – axis

(5) The image of the point $(1, 4)$ by transition two units downward is ....

(6) The image of the point $(-2, -1)$ by transition 3 units in the positive direction of y – axis ....

(7) The image of the point $(1, 4)$ by transition $(3, -4)$ ....

(8) The image of the point $(1, 2)$ by transition $(-3, 1)$ ....

(9) The image of the point $(5, 2)$ by transition $(x, y)\to(x+1, y-2)$ is ....

(10) The point $(-3, 1)$ is the image of the point $(4, 1)$ by transition ....

(11) The image of the point $(-3, 6)$ by transition $(x, y)\to(x, y-1)$ is ....

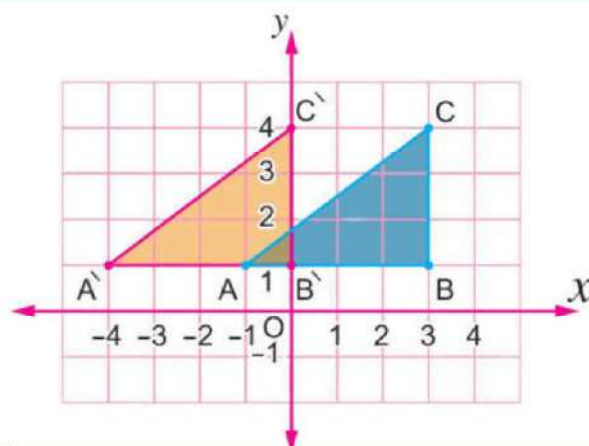
(12) The point $(3, -1)$ is the image of the point $(-3, 1)$ by transition ....

Example 4

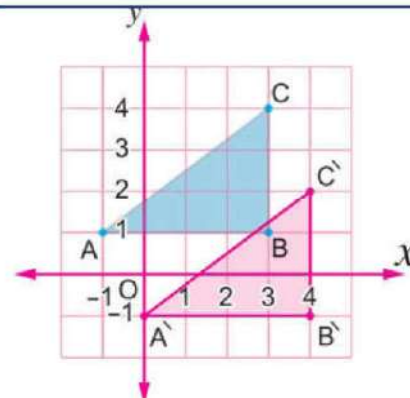
Draw the triangle ABC where $A(-1, 1)$, $B(3, 1)$, $C(3, 4)$ then :

(1) find its image by transition 3 units to the left

transition 3 units to the left equivalent the transition $(-3, 0)$

The point	Its image by transition 3 units to the left
$A(-1, 1)$	$A'(-1-3, 0)=(-4, 0)$
$B(3, 1)$	$B'(3-3, 1)=(0, 1)$
$C(3, 4)$	$C'(3-3, 4)=(0, 4)$

(2) find its image by transition $(1 , -2)$

Example 4

The point	Its image by transition $(1 , -2)$
$A(-1 , 1)$	$A'(-1+1 , 1-2) = (0 , -1)$
$B(3 , 1)$	$B'(3+1 , 1-2) = (4 , -1)$
$C(3 , 4)$	$C'(3+1 , 4-2) = (4 , 2)$

Third Rotation

⊙ Rotation is a geometric transformation that turns the geometric figure around a fixed point with a certain angle in a certain direction

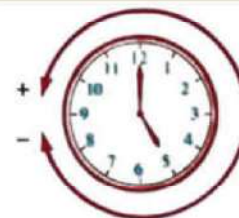
⊙ The point which the point rotates around is called the centre of rotation

⊙ The rotation is determined by the following elements :

(1) The centre of the rotation (2)The measure of the angle of rotation

(3) The direction of rotation

Notice that

(1) The measure of rotation is positive if the rotation is anticlockwise

(2) The measure of rotation is negative if the rotation is clockwise

(3) The rotation is symbolized by the symbol : $R(O , \theta)$ where O center of the rotation θ measuring the angle and direction of rotation

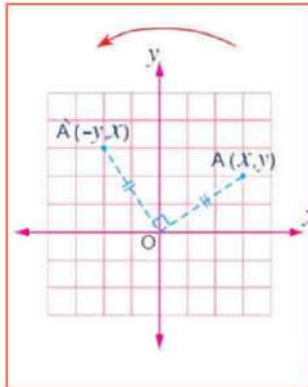
⊙ Rotating around the origin anticlockwise at an angle 90° written $R(O , 90^\circ)$

⊙ Rotating around the origin clockwise movement at an angle 90° written $R(O , -90^\circ)$

Rotation $R(O\ ,\ 90^\circ)$

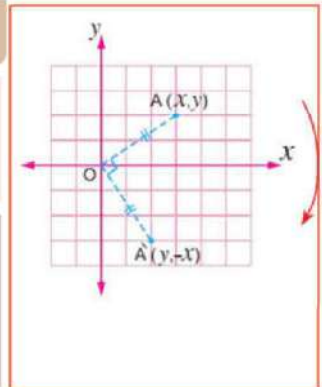
$A(x\ ,\ y) \xrightarrow{R(0,90^\circ)} A'(-y\ ,x)$

For example :

$(3\ ,5) \xrightarrow{R(0,90^\circ)} (-5\ ,3)$

Rotation $R(O\ ,\ -90^\circ)$

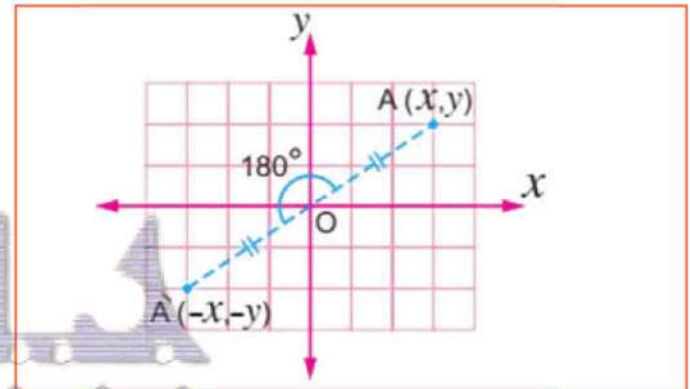
$A(x\ ,\ y) \xrightarrow{R(0,-90^\circ)} A'(y\ ,-x)$

For example :

$(-2\ ,3) \xrightarrow{R(0,90^\circ)} (3\ ,2)$

Rotation $R(O\ ,\ \pm180^\circ)$

$A(x\ ,\ y) \xrightarrow{R(0,\pm180^\circ)} A'(-x\ ,-y)$

For example

$(-2\ ,1) \xrightarrow{R(0,\pm180^\circ)} (2\ ,-1)$

Important remarks

(1) Rotation : $R(O\ ,\ 90^\circ)$ equivalent the rotation $R(O\ ,\ -270^\circ)$

(2) Rotation : $R(O\ ,\ -90^\circ)$ equivalent the rotation $R(O\ ,\ 270^\circ)$

(3) Rotation : $R(O\ ,\ 180^\circ)$ equivalent the rotation $R(O\ ,\ -180^\circ)$

(4) The rotation: $R(O\ ,\ \pm360^\circ)$ is called the identity rotation because it returns the figure to its original position

Example 5

Complete the following

(1) The image of the point $(1\ ,2)$ by rotation about the origin point with an angle of measure 90° is ....

(2) The image of the point $(2\ ,-3)$ by rotation about the origin point with an angle of measure -90° is ....

(3) The image of the point $(-2\ ,-3)$ by rotation about the origin point with an angle of measure -90° is ....

(4) The image of the point $(-1\ ,1)$ by rotation about the origin point with an angle of measure 90° is ....

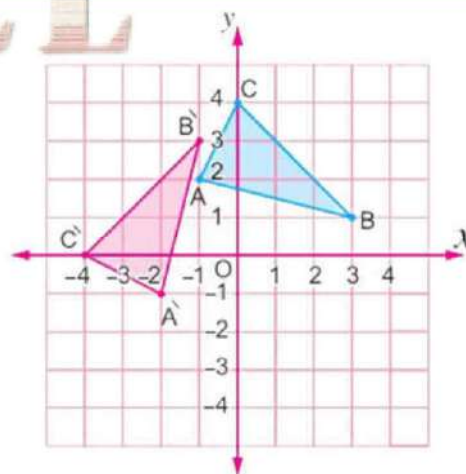
Example 5

(5) The image of the point $(0, 3)$ by rotation about the origin point with an angle of measure 90° is …….	(6) The image of the point $(-5, 0)$ by rotation about the origin point with an angle of measure -90° is …….
(7) The image of the point $(0, -4)$ by rotation about the origin point with an angle of measure -90° is …….	(8) The image of the point $(0, 7)$ by rotation about the origin point with an angle of measure -90° is …….
(9) The image of the point $(1, -3)$ by rotation about the origin point with an angle of measure 180° is …….	(10) The image of the point $(3, 4)$ by rotation about the origin point with an angle of measure -180° is …….
(11) The image of the point $(1, 3)$ by rotation about the origin point with an angle of measure 360° is …….	(12) The image of the point $(-1, 3)$ by rotation $(O, 270^\circ)$ is …….
(13) The image of the point $(-2, -5)$ by rotation $(O, -270^\circ)$ is …….	(14) The image of the point $(0, -5)$ by rotation $(O, -90^\circ)$ is …….
(15) The point $(-2, -5)$ is the image of the point $(5, -2)$ by rotation about the origin point with an angle of measure …………………..	(16) The point $(-2, 4)$ is the image of the point $(2, -4)$ by rotation about the origin point with an angle of measure …………………..

Example 6

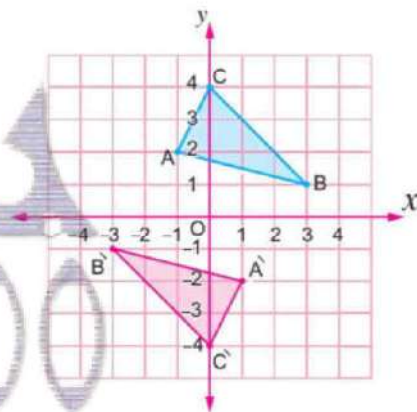
Draw the triangle ABC, where $A(-1, 2)$, $B(3, 1)$, $C(0, 4)$ then :

(1) Find its image by rotation $R(O, 90^\circ)$

The point	Image by rotation $R(O, 90^\circ)$
$A(-1, 2)$	$A'(-2, -1)$
$B(3, 1)$	$B'(-1, 3)$
$C(0, 4)$	$C'(-4, 0)$

(2) Find its image by rotation $R(O, -90^\circ)$

The point	Image by rotation $R(O, -90^\circ)$
$A(-1, 2)$	$A'(2, 1)$
$B(3, 1)$	$B'(1, -3)$
$C(0, 4)$	$C'(4, 0)$

(3) Find its image by rotation $R(O, \pm180^\circ)$

The point	Image by rotation $R(O, \pm 90^\circ)$
$A(-1, 2)$	$A'(1, -2)$
$B(3, 1)$	$B'(-3, -1)$
$C(0, 4)$	$C'(0, -4)$

Properties of reflection, transition and rotation

Reflection in a straight, transition, and rotation around a point in the plane maintains:

(1) Lengths of line segments (2) Angle measurements

(3) Parallelism (4) Interlayer

Home work — Geometrical transformation

Q1 Choose the correct answer

1 Which of the following is the image of the point $(5, 0)$ by reflection in the x axis

(a) $(5, 0)$ (b) $(-5, 0)$
(c) $(0, 5)$ (d) $(0, -5)$

2 Which of the following is the image of the point $(1, 3)$ by reflection in the x axis

(a) $(1, 3)$ (b) $(-1, -3)$
(c) $(1, -3)$ (d) $(3, -1)$

3 The image of the point $(-4, 2)$ by rotation about the origin point anticlockwise is ......

(a) $(-4, -2)$ (b) $(4, 2)$
(c) $(-2, 4)$ (d) $(-2, -4)$

4 Which of the following is the image of the point $(0, -3)$ by transition $(-1, 2)$

(a) $(-1, -1)$ (b) $(-1, 1)$
(c) $(1, -1)$ (d) $(1, 1)$

5 If the point A' is the image of the point A by reflection in the x – axis, where the point A lies in the third quadrant, then the point A' lies in the .....quadrant

(a) First (b) Second (C) Third (b) Fourth

6 Which of the following rotations makes the point $A'(x, -y)$ is the image of the point $A(-x, y)$

(a) $R(O, -90^\circ)$ (b) $R(O, 90^\circ)$
(c) $R(O, 180^\circ)$ (d) $R(O, 360^\circ)$

7 What is the rotation that makes the point is $A'(2, -6)$ the image of the point $A(-6, -2)$

(a) $R(O, -180^\circ)$ (b) $R(O, -90^\circ)$
(c) $R(O, 90^\circ)$ (d) $R(O, 180^\circ)$

8 What is the image of the point $(5, -2)$ by transition 5 units in the negative direction of the x – axis?

(a) $(5, -7)$ (b) $(10, -2)$
(c) $(0, -2)$ (d) $(5, -3)$

9 If the point $A'(4, -5)$ is the image of the point A by transition $(x, y) \to (x-2, y+1)$, then A = ...

(a) $(6, -4)$ (b) $(4, -4)$
(c) $(2, -4)$ (d) $(6, -6)$

10 What is the image of the point $(-1, 4)$ by transition $(3, -2)$

(a) $(2, 2)$ (b) $(-2, -2)$
(c) $(2, -2)$ (d) $(-2, 2)$

11

What is the image of the point $(3 , -5)$ by rotation $R(O , 180^\circ)$ ؟

(a) $(-3 , 5)$ (b) $(-3 , -5)$

(c) $(5 , 3)$ (d) $(5 , -3)$

12

What is the image of the point $(3 , -5)$ by rotation $R(O , 90^\circ)$ ؟

(a) $(-3 , 5)$ (b) $(-3 , -5)$

(c) $(5 , 3)$ (d) $(5 , -3)$

13

If the point $A'(2 , 5)$ is the image of the point A by transition $(x , y) \rightarrow (x , y-2)$ then : A ....

(a) $(2 , 3)$ (b) $(2 , 7)$

(c) $(0 , -2)$ (d) $(0 , 7)$

14

What is the image of the point $(2 , -1)$ by transition $(x , y) \rightarrow (x-3 , y+4)$?

(a) $(-1 , 5)$ (b) $(-3 , 4)$

(c) $(5 , 3)$ (d) $(-1 , 3)$

15

If the point $A'(x+1 , -2)$ is the image of the point $A(-4 , 2)$ by rotation $(O , 180^\circ)$ what is the value of x ?

(a) -1 (b) 3

(c) -2 (d) -5

16

What is the transition that makes the point $A'(-2 , 1)$ is the image of the point $A(4 , -5)$

(a) $(-6 , 6)$ (b) $(-6 , -4)$

(c) $(2 , -4)$ (d) $(6 , -6)$

17

If the image of a point by rotation about the origin point is it self , then the measure of the rotation angle is ....

(a) 90° (b) -90°

(c) 180° (d) 360°

18

Which of the following is the image of the point $(-2 , -5)$ by reflection in the y- axis ?

(a) $(2 , 5)$ (b) $(2 , -5)$

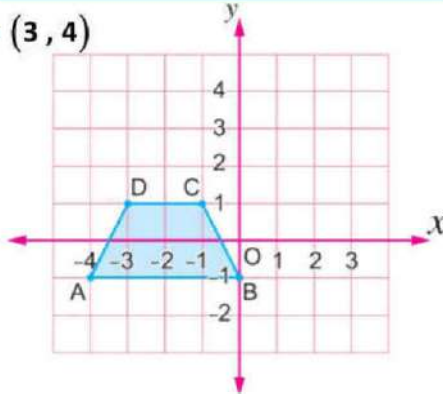
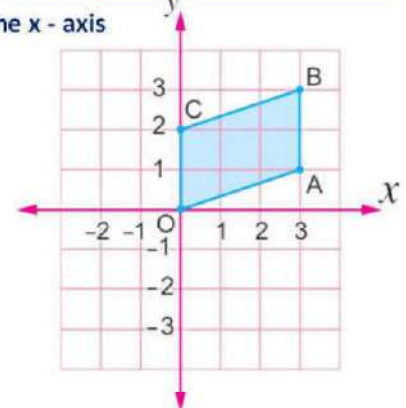
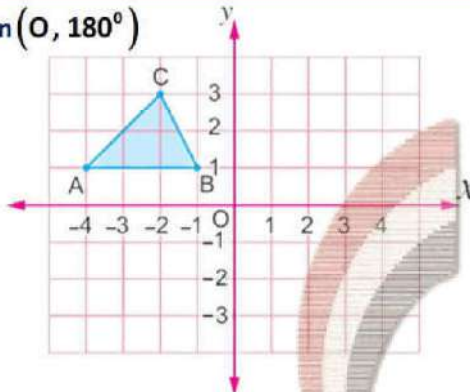
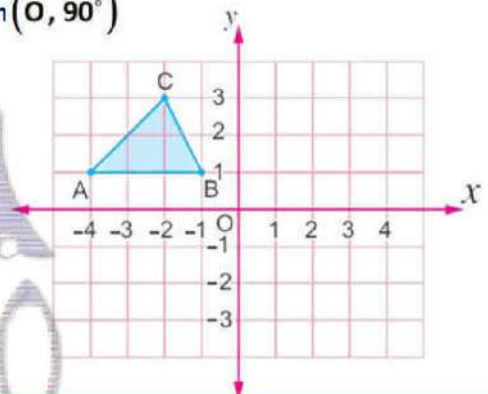
(c) $(-2 , 5)$ (d) $(-5 , -2)$

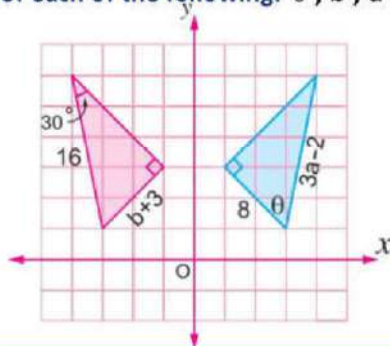
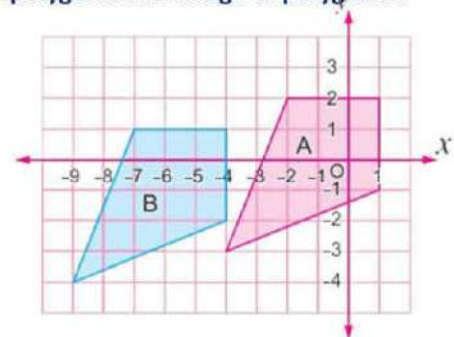
19

Which of the following represents the rotation of the opposite square around its center at an angle measured 90° clockwise

(a) (b) (c) (d)

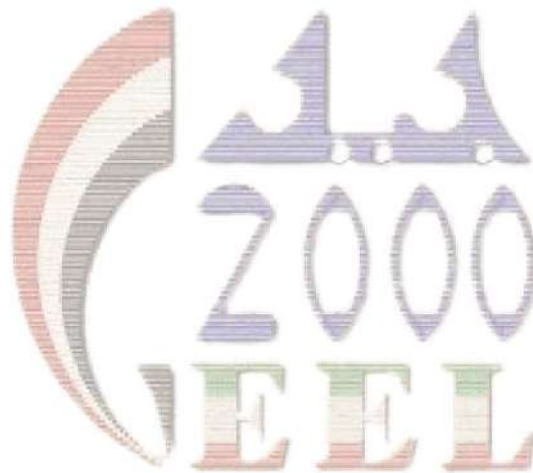
Q2 Draw the image of each of the following shapes using the geometric transformation mentioned next

1 Transition $(3, 4)$

2 Reflection on the x - axis

3 Rotation $(O, 180^\circ)$

4 Rotation $(O, 90^\circ)$

Q3 Answer the following questions

1 In the following figure, if one of the triangles is the image of the other by reflection in the y-axis, find the value of each of the following: θ , b , a

2 In the following figure, identify the translation that makes polygon A the image of polygon B.

3 draw $\overline{AB}$ where $A(3, 2)$, $B(-1, 1)$, then draw its image by the transition $(-2, -5)$

4 Draw Δ ABC where: $A(-6, 6)$, $B(-2, 2)$, $C(4, 1)$ then draw its image by $R(O, -90^\circ)$

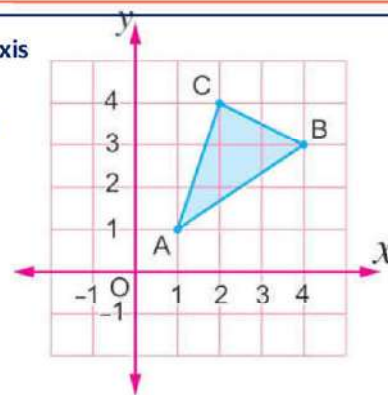
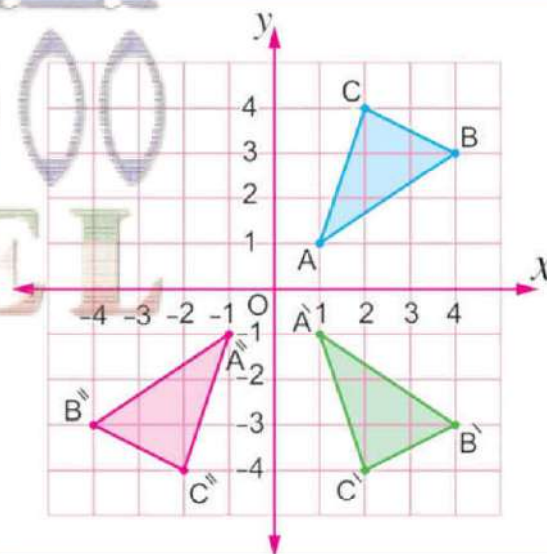
5	Draw ΔABC where: $A(1,1)$, $B(3,4)$, $C(1,1)$ then draw its image by reflection on the y – axis	6	Draw ΔABC where $A(1,1)$, $B(3,4)$, $C(1,1)$, then draw its image by reflection in the x – axis
7	Draw ΔABC where $A(1,1)$, $B(5,1)$, $C(5,6)$, then draw its image by the rotation $R(O, 180^{\circ})$	8	draw ΔABC where: $A(4,4)$, $B(4,2)$, $C(1,2)$ then draw its image by the transition $(x-2, y+1)$
9	Draw ΔABC where $A(1,1)$, $B(5,1)$, $C(5,6)$, then draw its image by the rotation $R(O, 270^{\circ})$	10	Draw ΔABC where: $A(0,2)$, $B(5,0)$, $C(6,4)$, then draw its image by the rotation $R(O, 90^{\circ})$

Lesson 12 **Composite Geometrical transformations**

Composite Geometrical transformations

It is the process of performing successive geometric transformations on a geometric shape, and sometimes the geometric shape resulting from the composition can be described by a single geometric transformation equivalent to this composition.

Example 1

Draw $\Delta A'B'C'$ the image of the triangle ΔABC by the reflection on the x – axis

, then draw $\Delta A''B''C''$ the image of $\Delta A'B'C'$ by the reflection on the y - axis

The point	Reflection on x-axis	Reflection on y-axis
$A(1,1)$	$A'(1,-1)$	$A''(-1,-1)$
$B(4,3)$	$B'(4,-3)$	$B''(-4,-3)$
$C(2,4)$	$C'(2,-4)$	$C''(-2,-4)$

Notice that

(1)A reflection in the X-axis followed by a reflection in the Y-axis is equivalent to a rotation $(O, 180^\circ)$

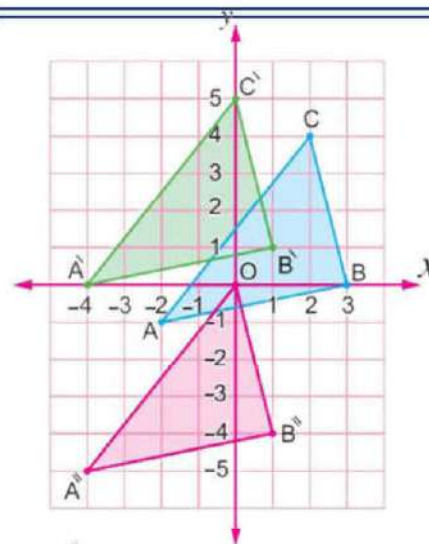
(2)Reflection in the X-axis followed by reflection again in the X-axis is the same .

(3)Reflection in the Y-axis followed by reflection again in the Y-axis is the same.

Example 2

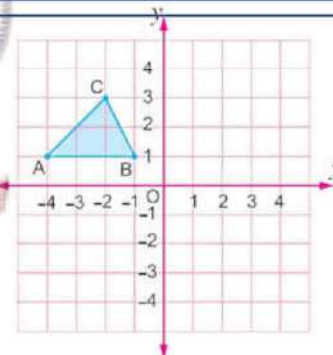
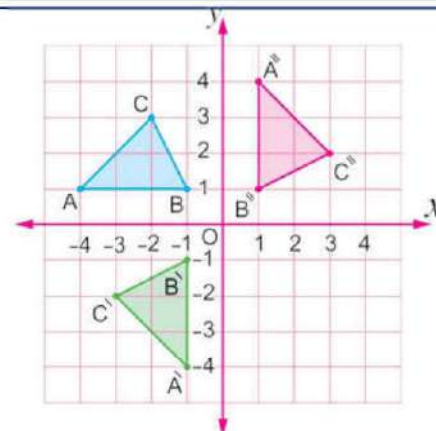
Draw ΔABC where $A(-2,-1)$, $B(3,0)$, $C(2,4)$ then draw its image by the transition $(-2,1)$ followed by the transition $(0,-5)$

The point	transition $(-2,1)$	transition $(0,-5)$
$A(-2,-1)$	$A'(-4,0)$	$A''(-4,-5)$
$B(3,0)$	$B'(1,1)$	$B''(1,-4)$
$C(2,4)$	$C'(0,5)$	$O(0,0)$

Notice that: transition $(-2,1)$ followed by the transition $(0,-5)$ equivalent the transition $(-2,1)+(0,-5)=(-2,-4)$

Notice that

⊙ Geometric transformation equivalent to a translation (a,b) followed by a transition (c,d) It is the transition $(a+c,b+d)$

Example 3

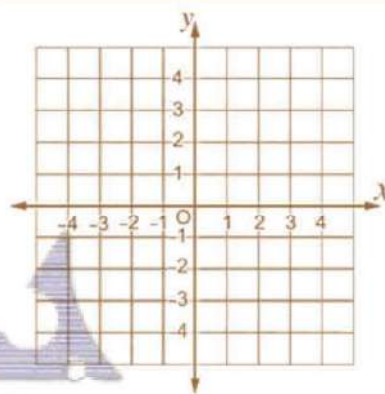
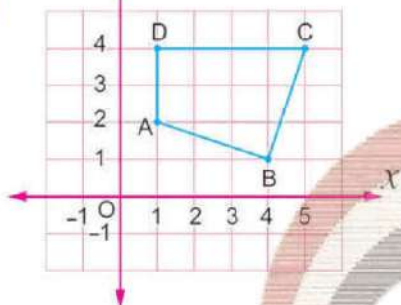
Draw the image of the triangle ABC by the rotation $R(O,90^\circ)$ followed by the rotation $R(O,180^\circ)$

The point	Rotation $R(O,90^\circ)$	Rotation $R(O,180^\circ)$
$A(-4,1)$	$A'(-1,-4)$	$A''(1,4)$
$B(-1,1)$	$B'(-1,-1)$	$B''(1,1)$
$C(-2,3)$	$C'(-3,-2)$	$C''(3,2)$

Notice that

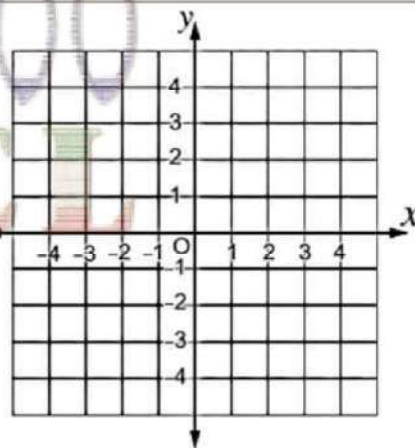
(1) Geometric transformation equivalent to rotation $R(O, 90^\circ)$ followed by rotation $R(O, 180^\circ)$ It is rotation $R(O, -90^\circ)$

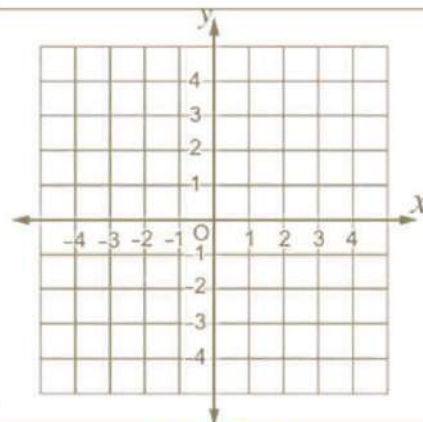
(2) Geometric transformation equivalent to rotation $R(O, 90^\circ)$ followed by rotation $R(O, 90^\circ)$ It is rotation $R(O, 180^\circ)$

Exercise

Answer the following questions

(1)Draw a picture of the polygon. ABCD by the transition $(1, -2)$ followed by the transition $(-3, 3)$

(2)Draw ΔABC where

egami sti ward neht $A(4, 4)$, $B(4, 1)$, $C(2, 0)$ by reflection on the x – axis followed by reflection on the y- axis

(3)Draw ΔABC where:

$A(1, 1)$, $B(3, 2)$, $C(1, 4)$, then draw its image by rotation $R(O, 90^\circ)$ followed by rotation $R(O, 180^\circ)$

Home work — Composite Geometrical transformations

Q1 Choose the correct answer

1 Geometric transformation equivalent to a translation $(1,3)$ followed by a transition $(0,2)$ It is a transition.......

(a) $(1,5)$ (b) $(-1,-1)$
(c) $(1,1)$ (d) $(0,5)$

2 The geometric transformation equivalent to a reflection in the X-axis followed by a reflection in the Y-axis is a rotation....

(a) $R(O,90^\circ)$ (b) $R(O,180^\circ)$
(c) $R(O,360^\circ)$ (d) $R(O,270^\circ)$

3 T h e i m a

(a) $(1,1)$ (b) $(1,7)$
(c) $(-1,7)$ (d) $(-1,5)$

4 What is the image of the point $(-2,1)$ by reflection on the x- axis followed by a reflection on the y - axis

(a) $(2,1)$ (b) $(-2,-1)$
(c) $(-1,-2)$ (d) $(2,-1)$

5 The rotation $R(O,90^\circ)$ followed by the rotation $R(O,180^\circ)$ equivalent to .....

(a) $R(O,-90^\circ)$ (b) $R(O,180^\circ)$
(c) $R(O,360^\circ)$ (d) $R(O,-270^\circ)$

6 T h e r o t

(a) $R(O,90^\circ)$ (b) $R(O,180^\circ)$
(c) $R(O,360^\circ)$ (d) $R(O,270^\circ)$

7 what is the image of the point $(-3,5)$ by reflection on x – axis followed by another reflection on x – axis

(a) $(3,-5)$ (b) $(-3,-5)$
(c) $(-3,5)$ (d) $(3,5)$

8 What is the image of the point $(2,-3)$ by reflection on the x – axis followed by reflection on the y – axis

(a) $(2,3)$ (b) $(-2,-3)$
(c) $(-2,3)$ (d) $(3,2)$

9 What is the image of the point $(-1,0)$ by transition $(1,0)$ followed by transition $(2,-3)$

(a) $(2,-3)$ (b) $(0,0)$
(c) $(1,0)$ (d) $(-1,0)$

10 W h a t i s

(a) $(-4,2)$ (b) $(4,2)$
(c) $(-4,-2)$ (d) $(4,-2)$

13 The image of the point $(-2, 3)$ by transition $(x, y) \rightarrow (x+1, y-2)$ followed by the transition $(-1, 2)$ is ....

(a) $(-2, 3)$ (b) $(2, 7)$

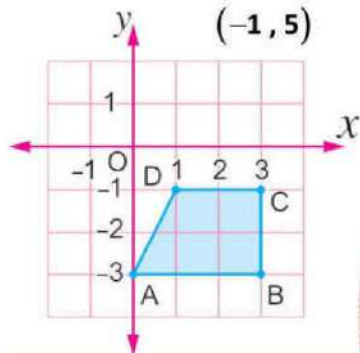
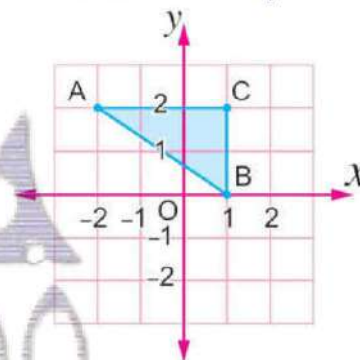
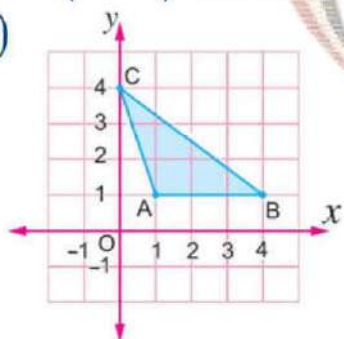
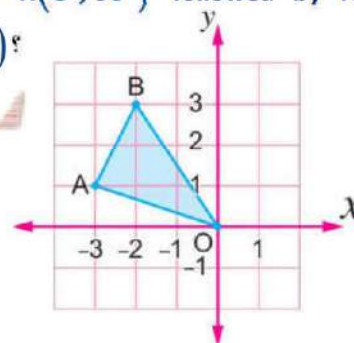
(c) $(0, -2)$ (d) $(0, 7)$

14 What is the image of the point $(-3, 0)$ by rotation $R(O, 90^\circ)$ followed by the rotation $R(O, -90^\circ)$ ؟

(a) $(3, 0)$ (b) $(0, 3)$

(c) $(0, -3)$ (d) $(-3, 0)$

Q2 **Draw the image of each of the following shapes using the geometric transformation mentioned next**

1 transition $(-1, 0)$ followed by the transition $(-1, 5)$

2 Reflection in X-axis followed by reflection in Y-axis

3 By rotation $R(O, 90^\circ)$ followed by rotation $R(O, 90^\circ)$ ؟

4 By rotation $R(O, 90^\circ)$ followed by rotation $R(O, -180^\circ)$ ؟

Q3 **Answer the following questions**

1 Draw in the coordinate plane $\overline{AB}$ where : $A(-1, 0), B(0, 3)$ then draw its image by rotation $R(O, 90^\circ)$ followed by rotation $R(O, 180^\circ)$

2 Draw the triangle ABC in the coordinate plane where: $A(1, 0), B(1, 4), C(3, 4)$ then draw its image by reflection in the x – axis followed by reflection on the y - axis

3 In the coordinates plane draw ΔABC where :

$A(2,1), B(2,5), C(5,6)$ then draw its image by rotation $R(O, 90^\circ)$ followed by rotation $R(O, 90^\circ)$ ؟

4 In the coordinate plane draw the square ABCD where :

$A(-1,0), B(3,0), C(3,4), D(-1,4)$ then draw its image by transition $(1,0)$ followed by the transition $(x, y) \rightarrow (x-2, y)$

5 In the coordinate plane draw $\overline{AB}$ where :

$A(-3,1), B(2,3)$ Then draw its image by rotation $R(O, -90^\circ)$ followed by rotation $R(O, -180^\circ)$ ؟

6 In the coordinate plane draw ΔABC where :

$A(0,1), B(5,2), C(5,5)$ then draw its image by transition $(1,3)$ followed by the transition $(-2,1)$

Lesson 13 Random experiment - sample space -

Random experiment

It is any experiment for which all possible outcomes can be known before it is conducted, but we cannot determine which of these outcomes will actually occur when it is conducted.

Sample space (outcome space)

⊙ It is the set of all possible outcomes of a random experiment.

⊙ The sample space is usually denoted by the symbol (S) ،The number of elements of the sample space is denoted by the symbol $n(S)$

Examples of

⊙ Experiment with tossing a regular coin once and noting the face that appears.

- A random experiment and its possible outcomes are the appearance of a Head. (H) Or Tail appears (T)
- sample space $S=\{H , T\}$
- Number of elements in the sample space $n(S)=2$

⊙ Experiment with throwing a regular die once and noting the number shown on the top face.

- A random experiment whose possible outcome is the appearance of one of the numbers $6 , 5 , 4 , 3 , 2 , 1$
- sample space $S=\{1 , 2 , 3 , 4 , 5 , 6\}$
- Number of elements in the sample space $n(S)=6$

Notice that

⊙ The experiment of choosing a card with the letter (A) from a group of identical cards that all have the letter (A) is not a random experiment because its result can be known with certainty before it is conducted, which is the letter (A).

⊙ The experiment of drawing a colored ball from a box containing a number of identical balls whose colors are unknown is not a random experiment because the color of the ball cannot be predicted before conducting the experiment.

Example 1

Determine which of the following experiments is random and which is not, then write the sample space for each of the random experiments, indicating the number of its elements.

(1)Drawing a ball from a bag containing a red ball, a blue ball, and a white ball, all of which are identical, and note their color.	⊙ Random experiment, assuming: red $=R$ ،Blue $=B$ White $=W$ ⊙ Simple space $S=\{R , B , W\}$ ⊙ $n(S)=3$

Example 1

(2)Drawing a card from 5 identical cards, all of which have the number 9 written on them, and note the number written on the card.	⊙ Not a random experiment
(3)Draw a card from 6 identical cards numbered from 2 to 7 and note the number written on the card.	⊙ A random experiment ⊙ $S=\{2,3,4,5,6,7\}$ ⊙ $n(S)=6$

Exercise

Determine which of the following experiments is random and which is not, then write the sample space for each of the random experiments, indicating the number of its elements.

(1)Drawing a ball from a bag containing a white ball, a yellow ball, a red ball, and a green ball. They are all identical and noticing their color	
(2)Throwing a regular cube once, the faces of which have the numbers from 30 to 35, and note the number appearing on the upper face.	
(3)Drawing a card from 7 identical cards, all of which have the number 5 written on them, and note the number written on the card.	

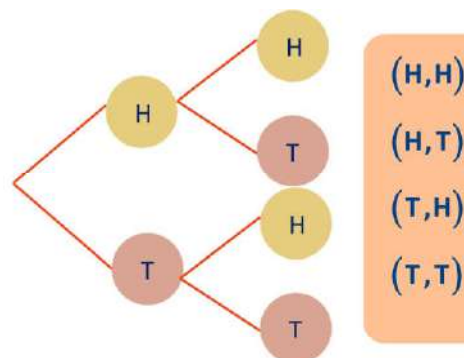
Example 2

.Write the sample space for each of the following two random experiments, indicating the number of its elements

(1)Experiment with tossing a regular coin twice in a row and observing the sequence of heads and tails appearing.

⊙ Each outcome of the experiment is an ordered pair, the first projection of which is the outcome of the first throw and the second projection of which is the outcome of the second throw. Since the possible outcomes of both the first and second throws are: the image (H) or the writing (T), the corresponding graph tree can be used to find the elements of the sample space.

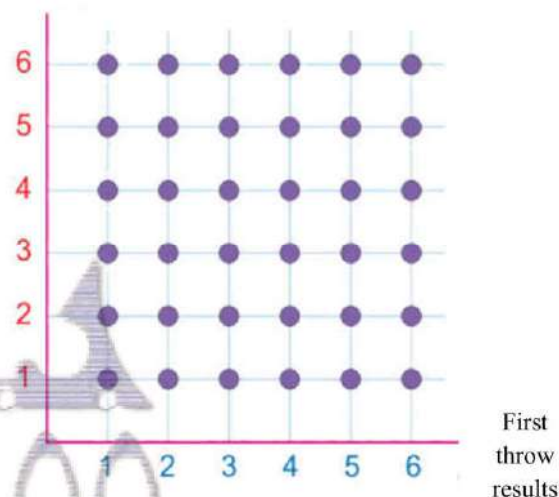
⊙ $S=\{(H,H),(H,T),(T,H),(T,T)\}$

⊙ $n(S)=4$

Example 2

(2)Experiment with throwing a regular die twice in a row and observing the number appearing on the upper face in both throws.

⊙ Each outcome of the experiment is an ordered pair, the first projection of which is the outcome of the first throw, and the second projection of which is the outcome of the second throw. The sample space (S) can be represented in the form of a table or geometrically on a graph as follows:

6	5	4	3	2	1	
(1,6)	(1,5)	(1,4)	(1,3)	(1,2)	(1,1)	1
(2,6)	(2,5)	(2,4)	(2,3)	(2,2)	(2,1)	2
(3,6)	(3,5)	(3,4)	(3,3)	(3,2)	(3,1)	3
(4,6)	(4,5)	(4,4)	(4,3)	(4,2)	(4,1)	4
(5,6)	(5,5)	(5,4)	(5,3)	(5,2)	(5,1)	5
(6,6)	(6,5)	(6,4)	(6,3)	(6,2)	(6,1)	6

$n(S) = 6 \times 6 = 36$ ⊙

Notice that

⊙ The outcome space of the experiment of tossing two distinct coins (different in color, shape, or size)... simultaneously is the same as the outcome space of the experiment of tossing the same coin twice in succession. Each outcome of the experiment is an ordered pair whose first projection is the heads of the first coin, and whose second projection is the heads of the second coin.

⊙ The sample space for the experiment of rolling a regular die twice in a row is the same as the sample space for the experiment of rolling two distinct dice once.

Events

⊙ **Event:** is a subset of the sample space.

⊙ **Occurrence of an event:** An event is said to have occurred if the outcome of the random experiment after it has been conducted is one of the elements of the set that makes up this event.

⊙ **Certain event** (S): It is an event that must occur when conducting a random experiment.

⊙ **The impossible event** $(\varnothing)$: It is an event that cannot occur when conducting a random experiment.

⊙ **simple event** :It is a subset of the sample space. (S) Contains only one element

Example 3

In the experiment of throwing a die once and observing the number appearing on the upper face, write the sample space and then find the following events, indicating which of them are simple, which are certain, and which are impossible .

(1)Event (A) is the event of the appearance of an even number.

(2) Event (B) is the event of the appearance of the number 5.

(3) Event (C) is the event of the appearance of a number greater than 6.

(4) Event (D) is the event of the appearance of a number less than 6.

⊙ $S = \{1, 2, 3, 4, 5, 6\}$

(1) $A = \{2, 4, 6\}$ (2) $B = \{5\}$ " simple event "

(3) $C = \varnothing$ " Impossible event " (4) $D = \{1, 2, 3, 4, 5, 6\}$ " certain event "

Exercise

In the experiment of choosing an integer from the numbers 1 to 20, write the sample space and then find each of the following events, indicating which of them is simple, which is certain, and which is impossible .

(1)A prime number appeared. (2) An even number appeared. (3) A number less than 12 appeared.

(4) The number 21 appeared. (5) The number 9 appeared. (6) A number divisible by 3 appeared .

(7)The occurrence of a perfect square number (8) A number appeared that satisfies the inequality. $X > 3$

(9) The occurrence of an odd prime number

Example 4

From the set of numbers $\{3, 4, 6, 7\}$ Create a number of two different digits. Write the sample space and then find each of the following events

(1)Event (A) is the event "the tens digit is odd"

(2)Event (B) is the event "the number is divisible by 4"

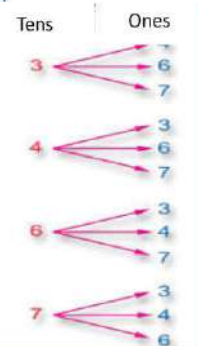
(3)Event (C) is the event "the sum of the two digits is 10"

⊙ $S = \left\{\begin{matrix} 34, 36, 37, 43, 46, 47, 63 \\ 64, 67, 73, 74, 76 \end{matrix}\right\}$

(1) $A = \{34, 36, 37, 73, 74, 76\}$

(2) $B = \{36, 64, 76\}$

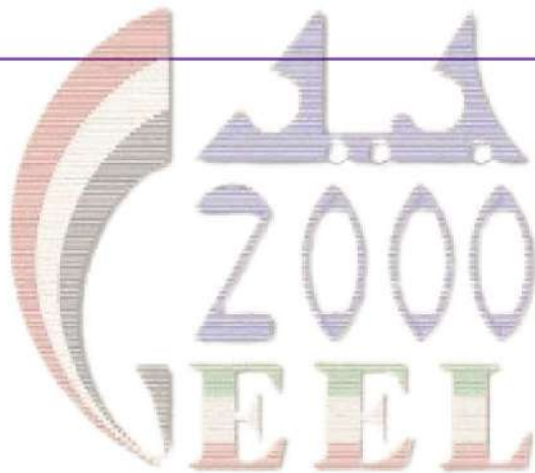
(3) $C = \{37, 46, 64, 73\}$

Exercise

In the experiment of throwing a regular die twice in a row and observing the number that appears on the upper face in the two throws, write each of the following events :

(1)Event (A) is the event of getting two numbers whose sum is 10.

(2)Event (B) is the event of getting two numbers whose smallest is 4.

(3)Event (C) is the event of getting two numbers whose largest is 3 .

(4)Event (D) is the event of obtaining two equal numbers .

Home work | Random experiment - sample space - events

Q1	Choose the correct amswer

1 In an experiment of choosing a digit of the number 5724 at random, what is the sample space?

(a) $\{2,4,5\}$ (b) $\{2,4,5,7\}$
(c) $\{57, 74, 42\}$ (d) $\{5742\}$

2 Drawing a card from a group of identical numbered cards without knowing the numbers written on the cards:

(a) Random experiment (b) not a random experiment
(c) Impossible event (d) Certain event

3 In the experiment of tossing a regular coin four times in a row, what is the number of elements in the sample space?

(a) 2 (b) 4
(c) 8 (d) 16

4 In an experiment to form a number from two different digits from a set of numbers $\{1, 3, 4\}$ How many elements of the event express that "the resulting number is an odd number"?

(a) 2 (b) 3
(d) 6 (c) 4

5 In the experiment of throwing a regular die once, which of the following events is a simple event?

(a) The occurrence of a number greater than 6.
(b) The occurrence of a prime even number
(c) The occurrence of a number less than or equal to 2.
(d) The occurrence of a prime odd number

Q2	Answer the following questions

1

In the experiment of throwing a regular die once and observing the number appearing on the upper face. Write the sample space and then write each of the following events, indicating which of them is simple, which is certain, and which is impossible:

(1)Event (A) is the event of a number greater than zero appearing.

(2) Event (B) is the event of a number appearing that is divisible by 3.

(3) Event (C) is the event of a number appearing that is less than or equal to 4.

(4) Event (D) is the event of a number appearing that satisfies the inequality: $5 > X$.

(5) Event (E) is the event of an odd, non-prime number appearing.

(6) Event (F) is the event of a number appearing that is greater than 4 and less than 5

2

A bag contains 25 identical cards numbered from 1 to 25. A card is drawn at random and the number shown on the drawn card is noted. Write down each of the following events :

(1)Event (A) is the event of a number appearing that is smaller than 4.

(2) Event (B) is the event of a number appearing that is a multiple of 6.

(3) Event (C) is the event of an odd number appearing that is divisible by 5.

(4) Event (D) is the event of a perfect square number appearing.

(5) Event (E) is the event of a perfect cube number appearing.

(6) Event (F) is the event of an even number appearing that is divisible by 3

3

In an experiment of tossing a regular coin twice in a row and observing the sequence of appearance of heads and tails. Write the sample space and then express each of the following events:

(1)Event (A) is the event: Heads appearing in the first toss.

(2) Event (B) is the event: Heads appearing in only one of the tosses.

(3)Event (C) is the event: The same thing appears in both tosses.

(4) Event (D) is the event: No heads appear.

(5)Event (E) is the event: Something different appears in the two tosses.

4

A regular coin is tossed, then a regular die. The top side of the coin and the number appearing on the top side of the die are observed. The sample space is represented by a tree. Then find the following two events:

(1) Event (A) is the event: "Earning a tail and an even number."

(2) Event (B) is the event: "Earning a head and an odd number".

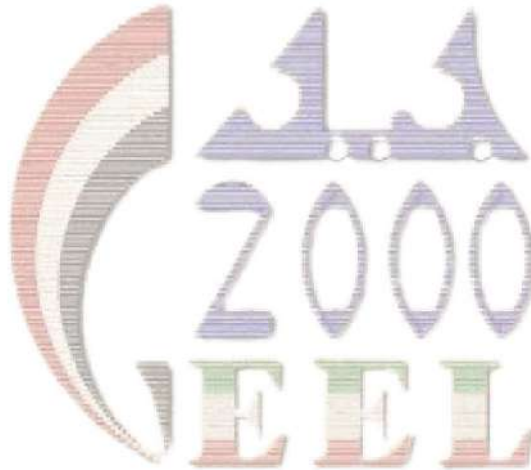
5

In the experiment of throwing a regular die twice in a row, write the following events:

(1)Event (A) is the event "the number 3 appears on the second throw"

(2)Event (B) is the event "the sum of two numbers appears greater than or equal to 10"

(3)Event (C) is the event "the sum of two numbers appears 15"

Lesson 14 Theoretical probability and experimental probability

Theoretical probability

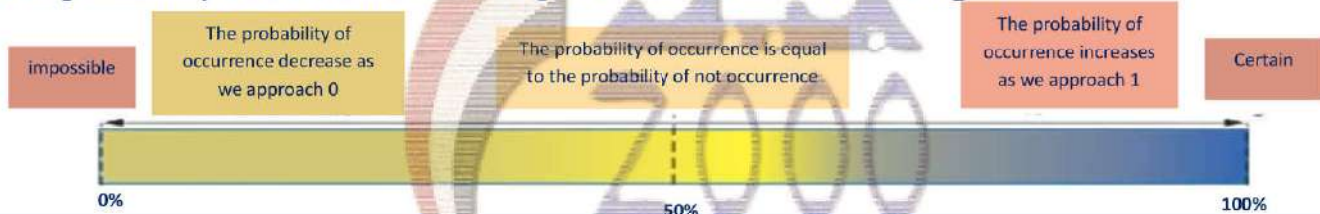
Theoretical probability is based on the principle of equal opportunity or equal potential, and is equal to the ratio between the number of outcomes of the event and the total number of outcomes.

Then : Probability of any event (A) : $P(A) = \frac{\text{Number of outcomes of event A}}{\text{Total number of outputs}}$

Notice that :

⊙ The probability of an impossible event is equal to zero and its written : $P(\varnothing) = \text{zero}$

⊙ The probability of a certain event is equal to one and its written : $P(S) = 1$

⊙ The probability of a certain event is equal to one as shown in the figure :

Answer the following questions

Example 1

(1)A card is drawn randomly from identical cards numbered from 1 to 12. Find the probability of each of the following events

(1) A event to get an even number

(2)B Event get prime number

(3) C event getting odd number greater than 7

(4) D event get whole square number

(5)E event get fewer than 2

⊙ $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

(1) The even numbers are : 2 , 4 , 6 , 8 , 10 , 12 $\therefore P(A) = \frac{6}{12} = \frac{1}{2}$

(2) The Prime numbers are : 2 , 3 , 5 , 7 , 11 $\therefore P(B)=\frac{5}{12}$

(3) The odd numbers greater than 7 are : 9 , 11 $\therefore P(C)=\frac{2}{12}=\frac{1}{6}$

(4) The whole square numbers are : 1 , 4 , 9 $\therefore P(D)=\frac{3}{12}=\frac{1}{4}$

(5) There is only number less than 2 and its: 1 $\therefore P(E)=\frac{1}{12}$

(2) When throwing a regular dice once and observing the number shown on the upper face, find the probability of each of the following events:

(1)A Event Get Odd Number

(2)B event getting fewer than 8

(3)C Event to get a prime odd number

(4)D Event Get Number 4

(5)E event get number greater than 6

(1) The Prime odd numbers are : 1 , 3 , 5 $\therefore P(A)=\frac{3}{6}=\frac{1}{2}$

(2) The numbers less than 7 are : 1 , 2 , 3 , 4 , 5 , 6 $\therefore P(B)=\frac{6}{6}=1$

(3) The prime odd numbers are : 3 , 5 $\therefore P(C)=\frac{2}{6}=\frac{1}{3}$

(4) 4 is only one number $\therefore P(D)=\frac{1}{6}$

(5) No numbers greater than 6 $\therefore P(E)=\frac{0}{6}=0$

(3) A box with 3 red balls, 7 white balls, 5 black balls, all the same, if a ball is randomly pulled from the box and its color is observed, what is the probability that the ball withdrawn:

(1)A red

(2)B blue

(3)C white or black

(4)D Not white

Let : red = R , White =W , black =B

Total number of balls $=5+7+3=15$

(1) $P(A)=\frac{3}{15}=\frac{1}{5}=0.2$ (2) $P(B)=\frac{0}{15}=0$

(3) $P(C)=\frac{7+5}{15}=\frac{12}{15}=\frac{4}{5}=0.8$ (4) $P(D)=\frac{3+5}{15}=\frac{8}{15}$

(4) A regular coin was thrown twice in a row and a sequence of pictures and writings was observed.

[1] A "Get Two Photos" event

[2] B "Get at least one image" event

[3] C happened "getting the same thing both times"

[4] D event "Get a picture in the second throw"

⊙ $S=\{(T,T),(T,H),(H,T),(T,T)\}$

[1] The products with two images are (H,H) and their numbers are 1

$\therefore P(A)=\frac{1}{4}$

[2] Outputs with at least one image are $(H,H),(H,T),(T,H)$ and their numbers are 3

$\therefore P(B)=\frac{3}{4}$

[3] The products with the same thing in the two throws $(H,H),(T,T)$ and their numbers are 2

$\therefore P(C)=\frac{2}{4}=\frac{1}{2}$

[4] The products with an image in the first throw $(H,H),(H,T)$ and their numbers are 2

$\therefore P(D)=\frac{2}{4}=\frac{1}{2}$

Notice that :

⊙ The sum of the probabilities of all outcomes of any randomized experiment =1

⊙ For any event A : $P(A) + notP(A) = 1$

Exercise

Answer the following questions

(1) A bag containing 5 black balls, 4 red balls, 3 blue balls, all the same. What is the probability that the ball drawn is:

[1] A black
[2] B white
[3] C red or blue
[4] D not blue

(2) A card is drawn randomly from identical cards numbered from 5 to 25. Find the probability of each of the following events

[1] A event to get an even number
[2] B Event to get an odd prime number
[3] C event getting an odd number greater than 20
[4] D event get whole square number
[5] E event get a number less than 2

(3) A regular die was shed once and the number shown on the upper face was observed Find the probability of each of the following events

[1] A prime-number appearance event
[2] B prime odd number appearance event
[3] C A number divisible by 3 occurred
[4] D The number 7 has appeared

Exercise

[5] E The occurrence of a number greater than 6 occurred.

(4)A regular coin was tossed twice in a row and the sequence of heads and tails was observed. Find the probability of each of the following events:

[1] A "Get Two Heads" event

[2] B Event "Get at least one Tail"

[3] C "Getting the same thing twice" event

[4] D Event "Get a head on the first throw"

Experimental probability

Experimental probability depends on conducting a practical experiment, then recording its results, and then using these results to calculate the probability as follows:

$$\text{Experimental probability for an event } (A) = \frac{\text{Number of times the experiment was performed}}{\text{Number of times event A occurs}}$$

Example 1

If a regular coin is tossed 100 times and heads appear 52 times, find the experimental probability of :

(1)Head (H) (2) Tails (T)

(1)The number of times the Head (H) appears is 52 times.

$P(H) = \frac{52}{100} = 0.52 = 52\%$

(2)The number of times the Tail (T) appears is: $100 - 52 = 48$

$P(T) = \frac{48}{100} = 0.48 = 48\%$

Example 2

A spinning disc is divided into several colored sectors of equal area. If the disc is spun 50 times and the corresponding table shows the number of times the pointer stops on each color.

(1)Find the experimental probability of the pointer stopping on yellow.

(2)Find the theoretical probability of the pointer stopping on yellow.

(3)If the number of times the disc is spun increases to 500 times, what do you expect about the chance of the pointer stopping on yellow?

the color	Number of appearances
Red	8
Blue	9
Yellow	13
Green	9
purple	11

Example 2

(1) The experimental probability of the indicator stopping on the yellow color. $\frac{13}{50} = 0.26 = 26\%$

(2) ∵The five colors are evenly distributed on the rotating disc.

∴ The theoretical probability of the indicator stopping on yellow $\frac{1}{5} = 0.2 = 20\%$

(3)When the number of times the disc is rotated increases to 500, we expect that the chances of the pointer stopping on the yellow color decrease until the experimental probability value approaches the theoretical probability value. 20%

Example 3

A regular die was thrown 150 times and the number shown on the top face was noted. The results of the numbers appearing were as follows:

The number	Number of appearances
1	28
2	19
3	23
4	28
5	25
6	27

⊙ Find the experimental probability:

(1)For the appearance of the number 2
(2)For not showing the number 5

⊙ Find the theoretical probability of getting the number 2.

⊙ (1) The number of times the number 2 appears is 19.

The experimental probability of getting the number 2 is $= \frac{19}{150}$

(2) The experimental probability of the number 5 not appearing.

$\frac{5}{6} = \frac{125}{150} = \frac{150-25}{150} =$

Or $\frac{5}{6} = \frac{125}{150} = \frac{28+19+23+28+27}{150} =$

⊙ Theoretical probability of getting the number 2 $\frac{1}{6} =$

Home work | **Theoretical probability - experimental probability**

Q1 **Choose the correct answer**

1 In an experiment of throwing a regular die once, what is the probability of getting a number that is divisible by 2?

(a) 0 (b) $33\frac{1}{3}\%$ (c) 50% (d) 75%

2 If you are considering buying one pen from a set of identical pens containing 5 red pens, 2 blue pens, and 3 black pens and you choose a pen at random, what is the probability that the pen is blue ?

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{2}{15}$ (d) $\frac{1}{15}$

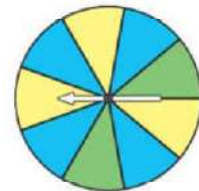
3 If event A is an event from a random experiment with equal chances of occurring, and the probability of event A is equal 40% to and the number of elements in the sample space is 15 elements, what is the number of elements in event A

(a) 2 (b) 4 (c) 6 (d) 10

4 When a regular die is thrown 10 times in a row, if the number 4 appears twice on the top face of the die, what is the experimental probability of not getting the number 4?

(a) $\frac{1}{6}$ (b) $\frac{2}{10}$ (c) $\frac{5}{6}$ (d) $\frac{8}{10}$

5 Hamza has a spinning toy divided into 9 equal sections. When it spins, the pointer randomly lands on one of the sections. What is the probability that the pointer lands on the blue or yellow color?

(a) $\frac{2}{9}$ (b) $\frac{4}{9}$ (c) $\frac{7}{9}$ (d) $\frac{8}{9}$

6 If a regular die is thrown once, what is the probability of getting an odd prime number ?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

7 What is the sum of the probabilities of all possible outcomes of a random experiment?

(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) 2

8 A letter is chosen from the group of letters of the word “school.” What is the probability that the letter is “S?”

(a) $\frac{1}{5}$ (b) $\frac{1}{6}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

9 If a regular coin is tossed once, the probability of getting a head is............

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) 1 (d) $\frac{2}{3}$

10 A card is drawn at random from cards numbered from 1 to 10. What is the probability of drawing a card with an odd number?

(a) $\frac{3}{10}$ (b) $\frac{4}{10}$ (c) $\frac{5}{10}$ (d) $\frac{7}{10}$

11 A bag contains 3 red balls and 4 white balls. If one ball is drawn at random, what is the probability that the drawn ball is white?

(a) $\frac{3}{4}$ (b) 1 (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

12 The probability of the certain event is equal to....

(a) 0 (b) 1 (c) 1− (d) ∅

13 A garden has five doors numbered from 1 to 5. What is the probability of someone leaving gate 2?

(a) $\frac{1}{5}$ (b) 1 (c) $\frac{2}{5}$ (d) $\frac{3}{4}$

14 If a student has a probability of his success of 0.8, what is the probability of his failing?

(a) 0.02 (b) 0.2

(c) 0 (d) 1

15 If a student has a 75% probability of his success, what is the probability of his failing?

(a) %25− (b) %15

(c) %125 (d) %25

16 A card was drawn randomly from a box with 35 identical balls, of which 7 were white balls and the rest were red and black in color, so what is the probability that the drawn ball is not white?

(a) $\frac{1}{35}$ (b) $\frac{1}{5}$ (c) $\frac{4}{5}$ (d) $\frac{34}{35}$

17 In the experiment of rolling a regular die once, what is the probability of appearing a number fewer than 5?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{6}$

Q2 Complete the following

1. The probability of occurring the impossible event = ...

2. The probability of occurring the certain event = ...

3. If a coin is flipped once , then the probability of appearance of head = ...

4. 10 cards numbered from 1 to 10 .If a card is drawn randomly , then the probability that the card in numbered by an odd number is ....

5. A box has 5 white balls , 7 red balls and 3 blue balls .If a ball is drawn randomly from the box , then the probability that the ball is blue = ...

6. In the experiment of throwing a fair die once and observing the upper face , the probability that the appearance number is less than 1= ....

7. A box contains 48 oranges , 4 of them are bad .If an orange is drawn randomly , then the probability that the drawn orange is good = ...

8. If the probability of occurring an event is 5 8 , then the probability that the event doesn't occur = ...

Q3	Answer the following questions

1

A classroom with 15 students, 4 of whom have black hair, 5 have brown hair, 6 have yellow hair, if selected is a random student, find the probability that the student is:

(1) His hair is black (2) His hair is not brown. (3) Yellow or brown

2

In the experiment of rolling a regular die once, find the probability of each of the following events:

(1) A is the appearance of a number more than 4 (2) B is the appearance of and even number

(3) C is the appearance of a number more than 4 (4) D is the appearance of the number 2

(5) E is the appearance of a number less than 3 (6) F is the appearance of one of the factors of the number 6

3

A card is chosen randomly from cards numbered from 20 to 29, What is the probability that the chosen card shows :

(1) A number greater than 25 (2) A number less than 25

(3) a prime number (4) an even number

4

If you randomly choose one number from the set of numbers $\{31, 29, 23, 19, 17, 13\}$

Find the probability that the sum of the numbers of the chosen number is an even number

5

In the experiment of forming a two-digit number from a set of numbers $\{4, 5, 7\}$, what is the probability of each of the following events:

(1) A occurrence the sum of the two digits 9

(2) B occurrence the number of tens odd

(3) C occurrence the product of the two numbers 35

(4) D occurrence the number tens = the number of ones

6

A bag containing 40 identical marbles, if Hani draws one marble randomly and finds it red, and the probability of dragging a red ball is equal to $\frac{3}{5}$ Find the number of red wear and tear in the bag

7

From the set of numbers $\{3, 4, 5\}$ Form a two-digit number and then find the probability of each of the following two events:

(1) A happened to be the number of ones odd (2) B happened to be the sum of the two digits 8

60

Khaled has a spinning game divided into 8 equal sections, when you spin the cursor randomly falls on one of the sections Find each of the following:

(1) Probability that the indicator falls on a number greater than or equal to 4

(2) The probability that the indicator falls on a number divisible by 6